David Lewis Meets Arthur Prior Again

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Inferential Mechanisms and
their Linguistic Manifestation
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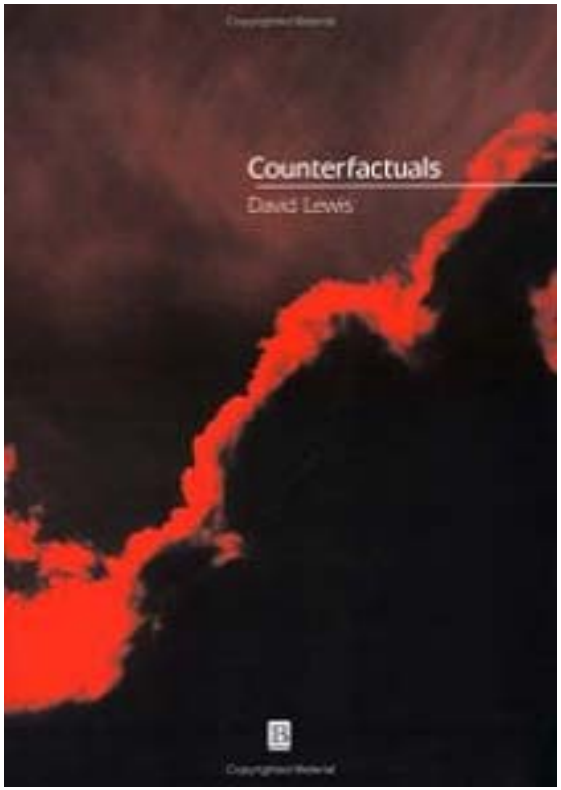
A Short Intellectual History of This Talk

- I'm a modal logician.
- I took Stefan's three-day tutorial on Modality in 2005 at Kyoto university.
- At that time, I first knew Kratzer's semantics based on modal base and ordering source.
- I wanted to know about some logical property of that semantics.
- Stefan recommended me to read Lewis (1973).
- Independently, my supervisor also recommended me to read it.

Counterfactuals



D. Lewis
Counterfactuals
Blackwell Publishing, 1973

A Short Intellectual History of This Talk (Cont.)

- I organized a reading group for Lewis (1973) with some students of Takubo-sensei and my colleagues.
- When I read through it, I found an interesting connection with my own research and wrote up:

  K. Sano
  Hybrid Counterfactual Logic
  Journal of Logic, Language and Information, 18, pp.515-539, 2009.
- After this study, I began to be interested in the interaction between logic and formal semantics.

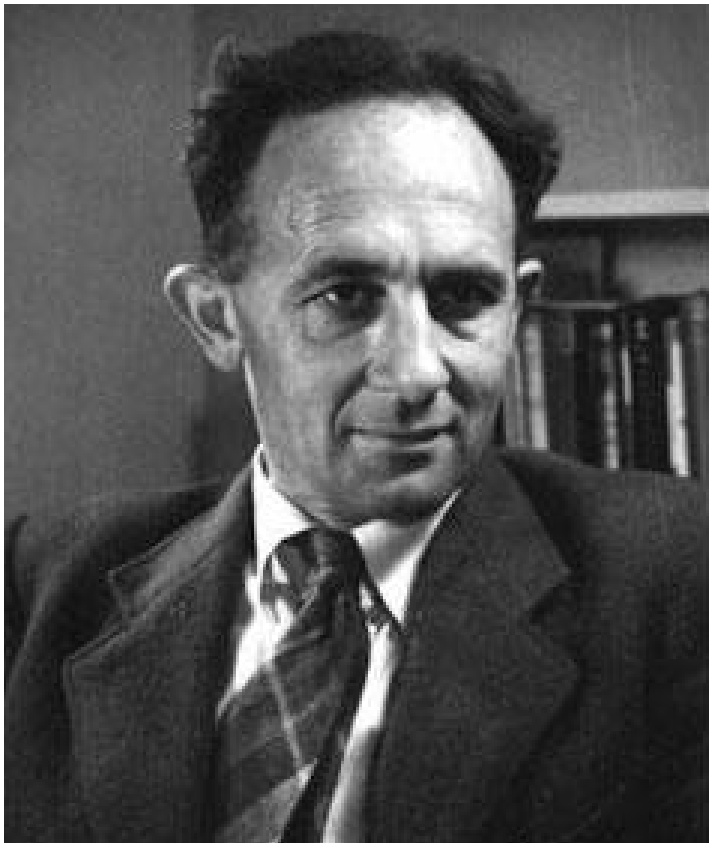
Outline

1. A Brief Introduction to Hybrid Logic
2. David Lewis' Counterfactual Logic
3. Hybrid Counterfactual Logic

Modal Formalism

- p is true at w:
  - p holds at the world w.
  - p holds at the point of time w.
  - The individual w has the property p.
- $\Box p$ is true at w:
  - p is true at any possible worlds relative to w.
  - p is true at any points of time later than w.
  - Any individuals related to w have the property p.
    - [Child]Happy: Any children of w are happy.

Fathers of Hybrid Logics



Prior

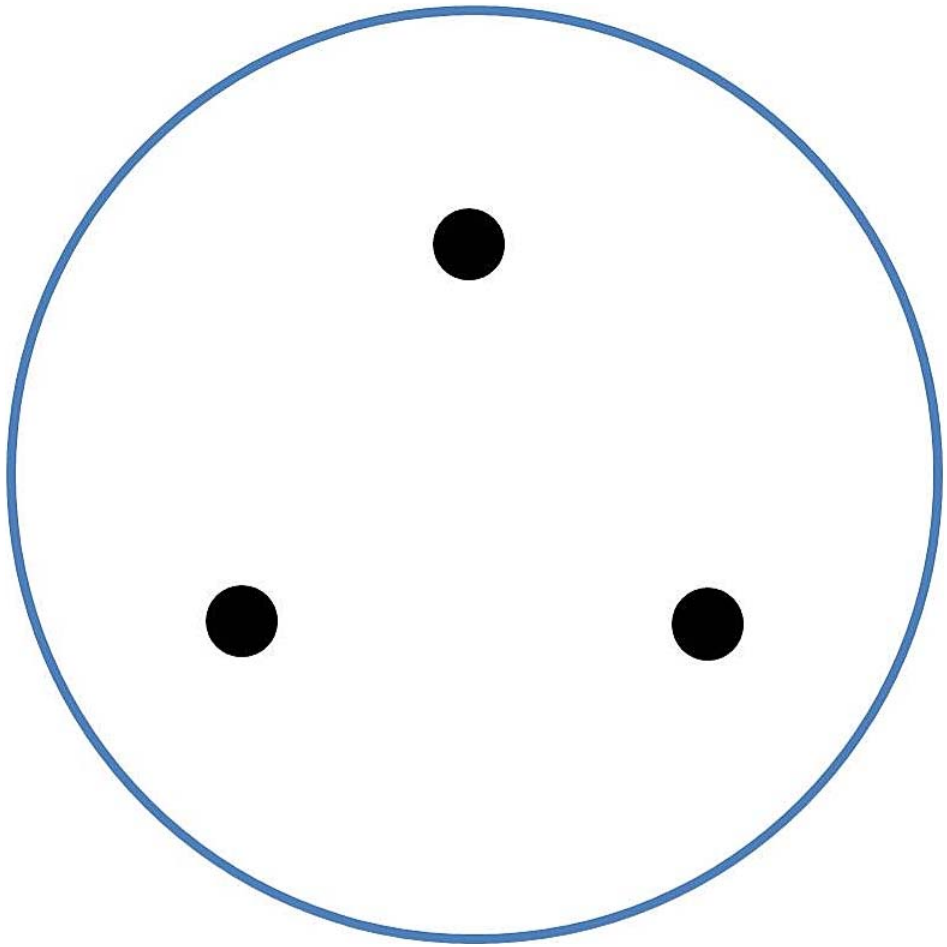
Gargov-Passy-Tinchev

Naming Points

Hybrid Formalism: Nominals and Satisfaction Operators

- A nominal i is true at w iff: i is a name of w.
  - time: 9 : 00, 5th June 2010, 2010, etc.
  - individual: TARO, HANAKO, etc.
- $@_i p$ is true at w iff: p is true at the world named by i.
  - $@_{11:00}$(Taro runs): Taro runs at 11:00.
  - $@_{\mathrm{HANAKO}}$Run: Hanako runs.

Semantics for Nominals and Satisfaction Operators



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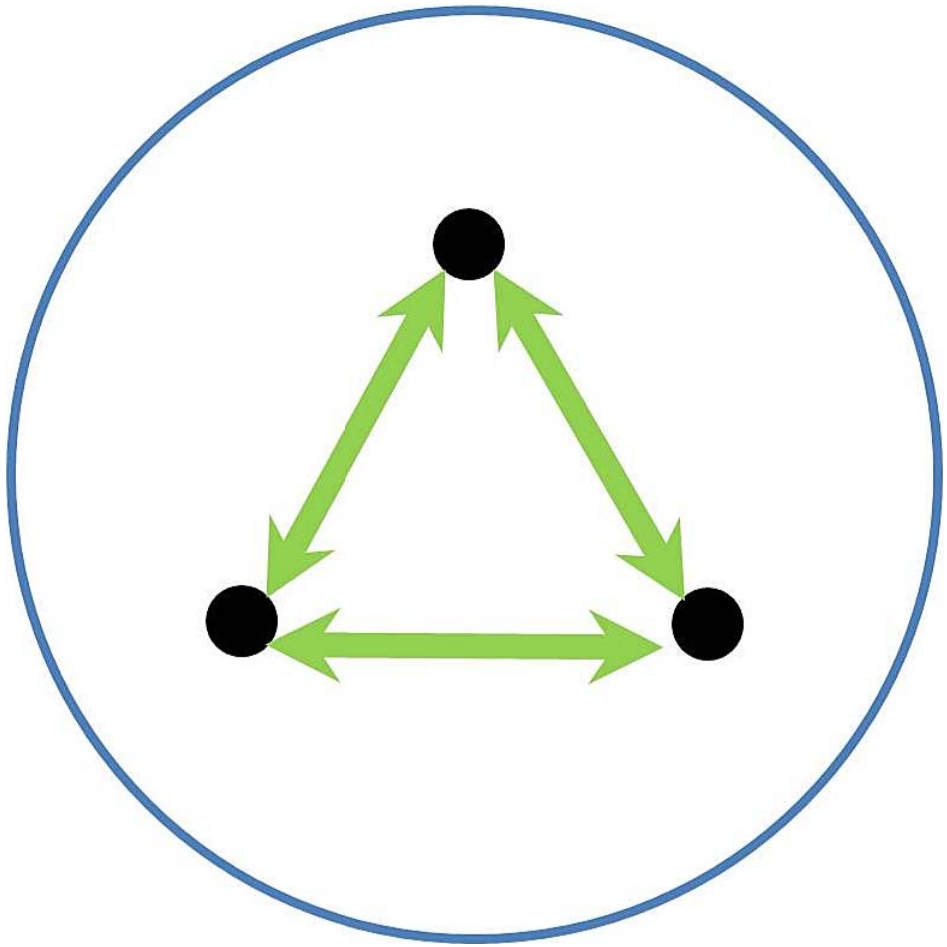

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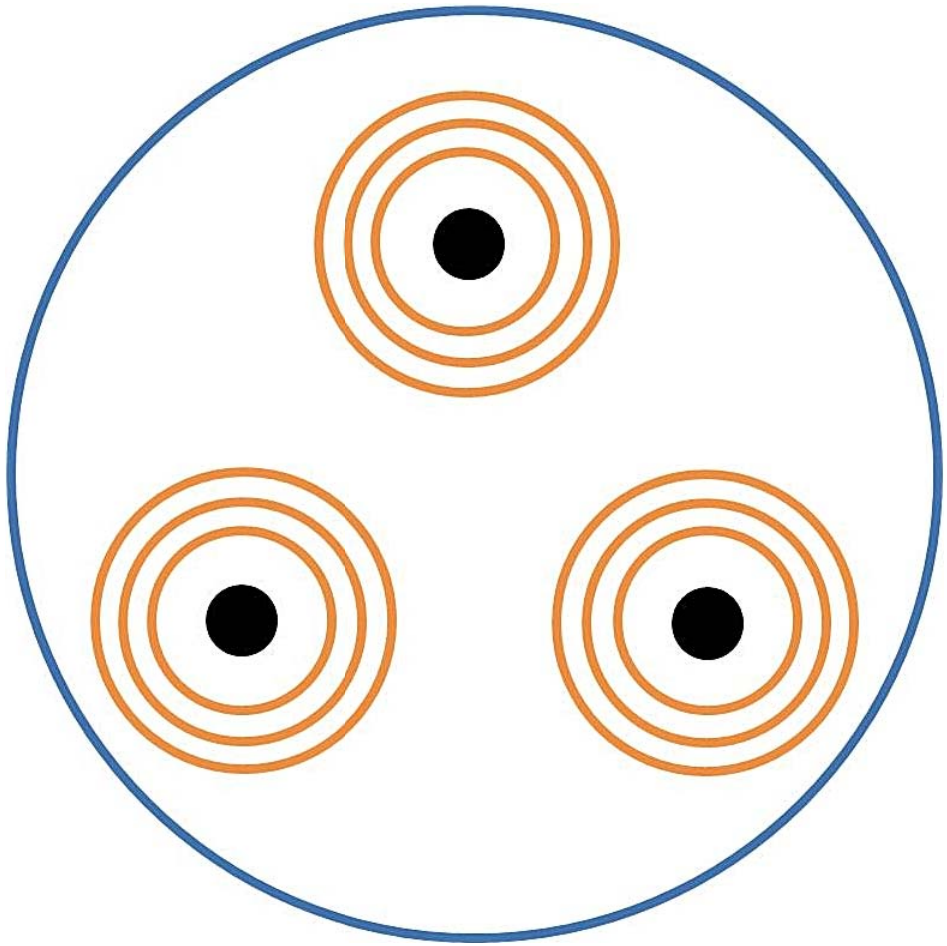

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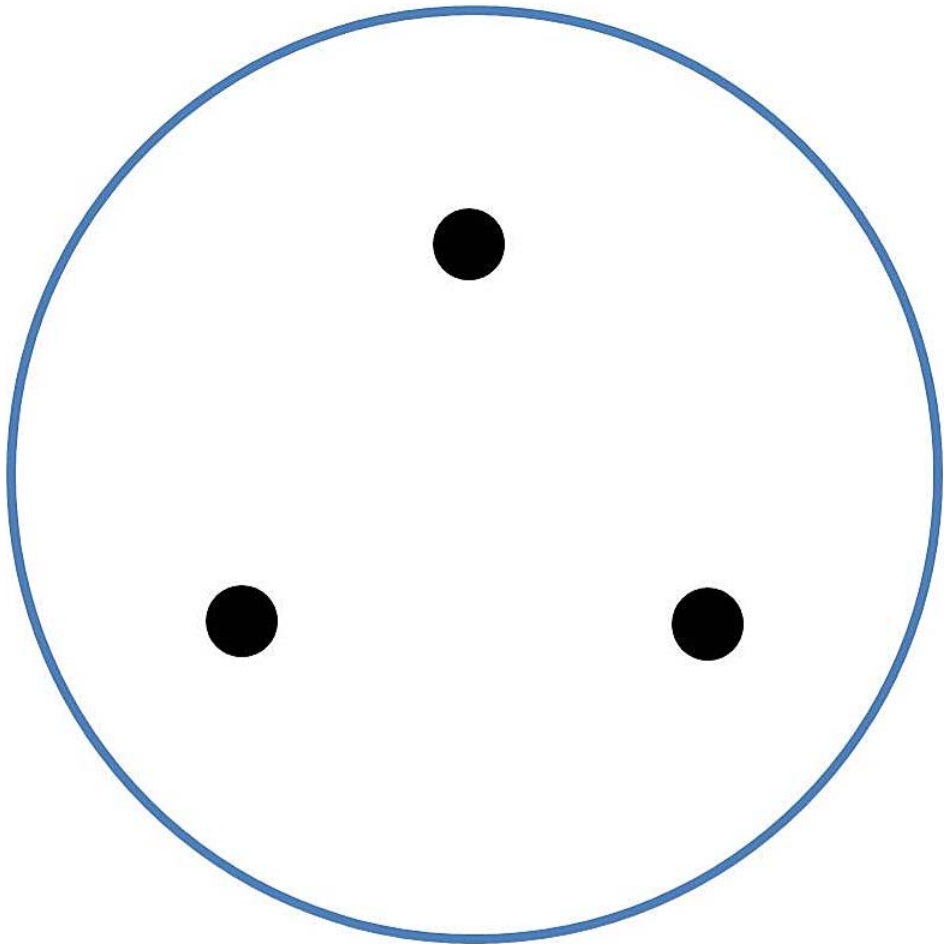

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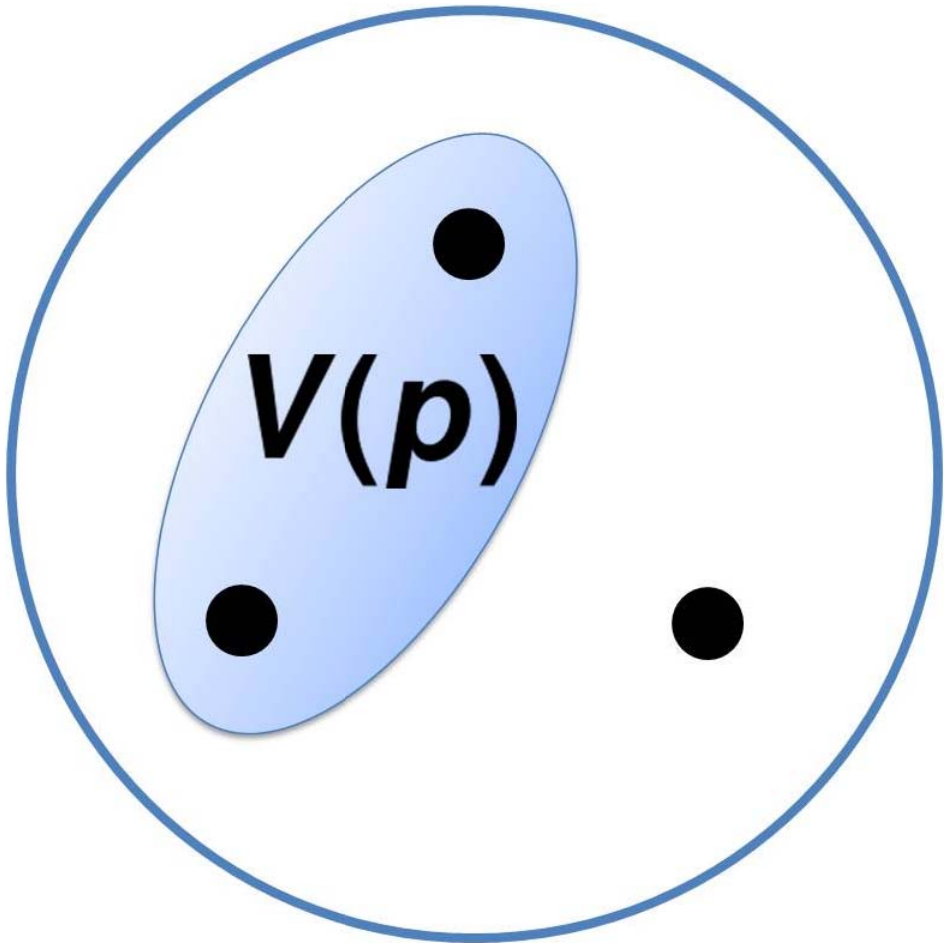

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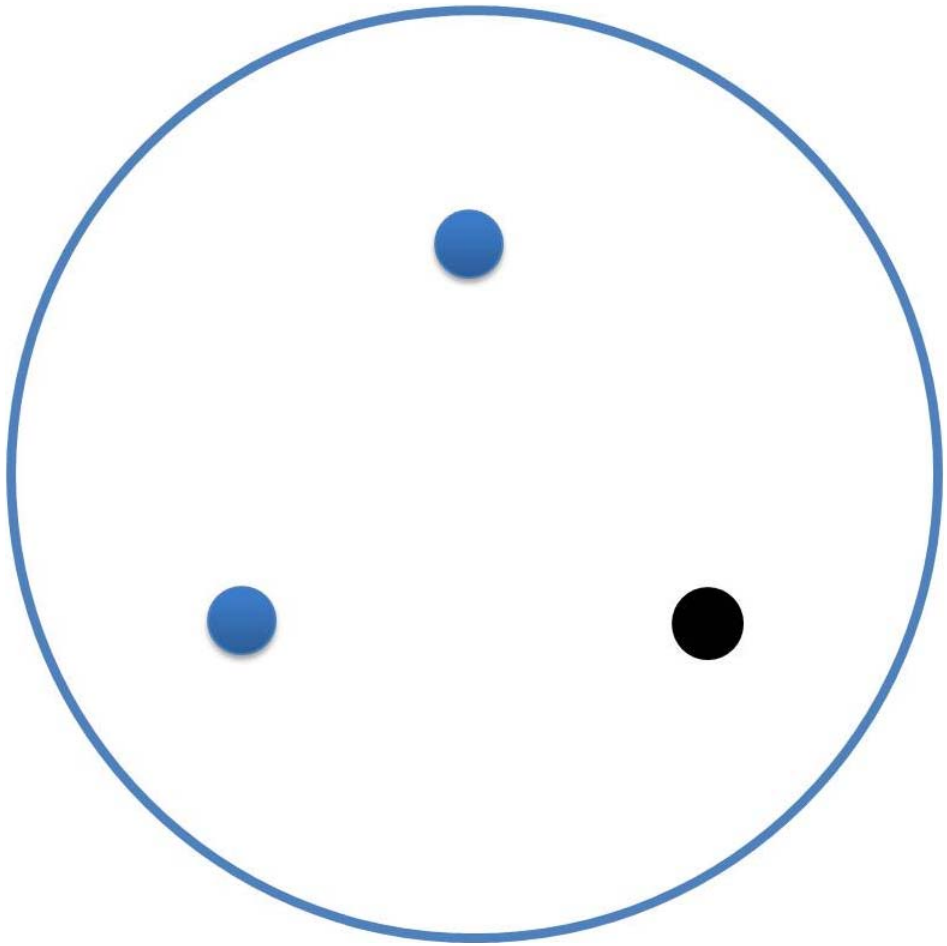

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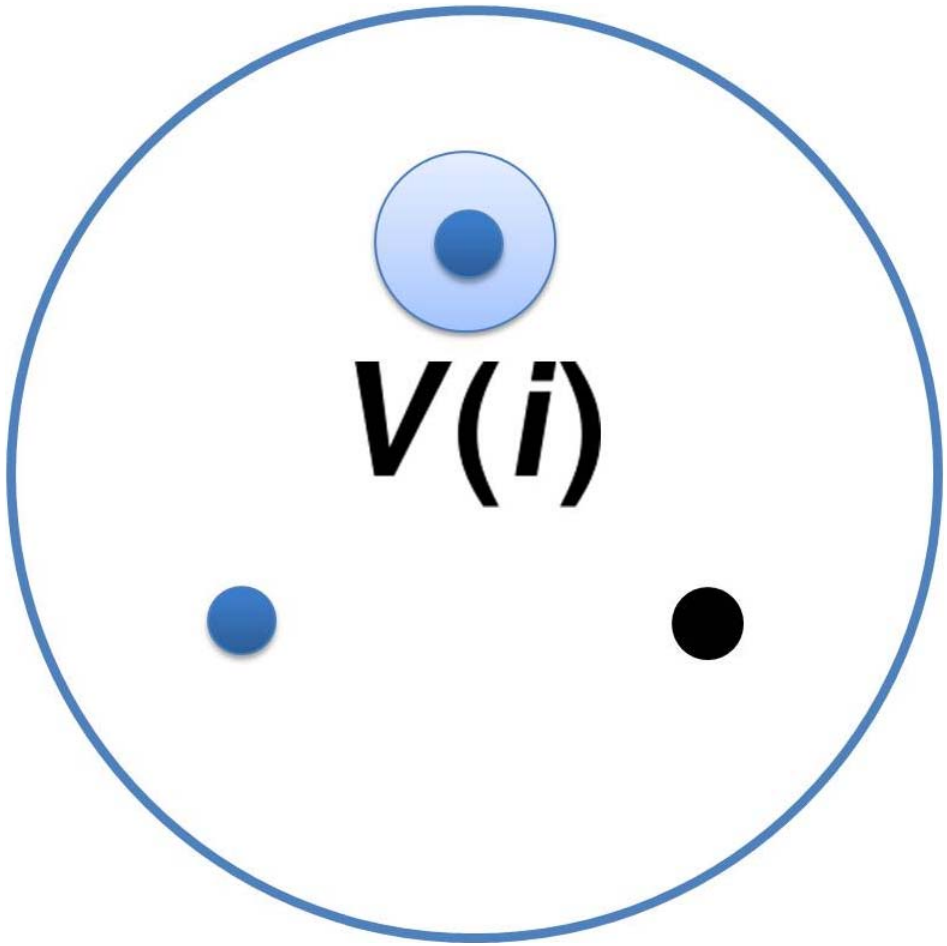

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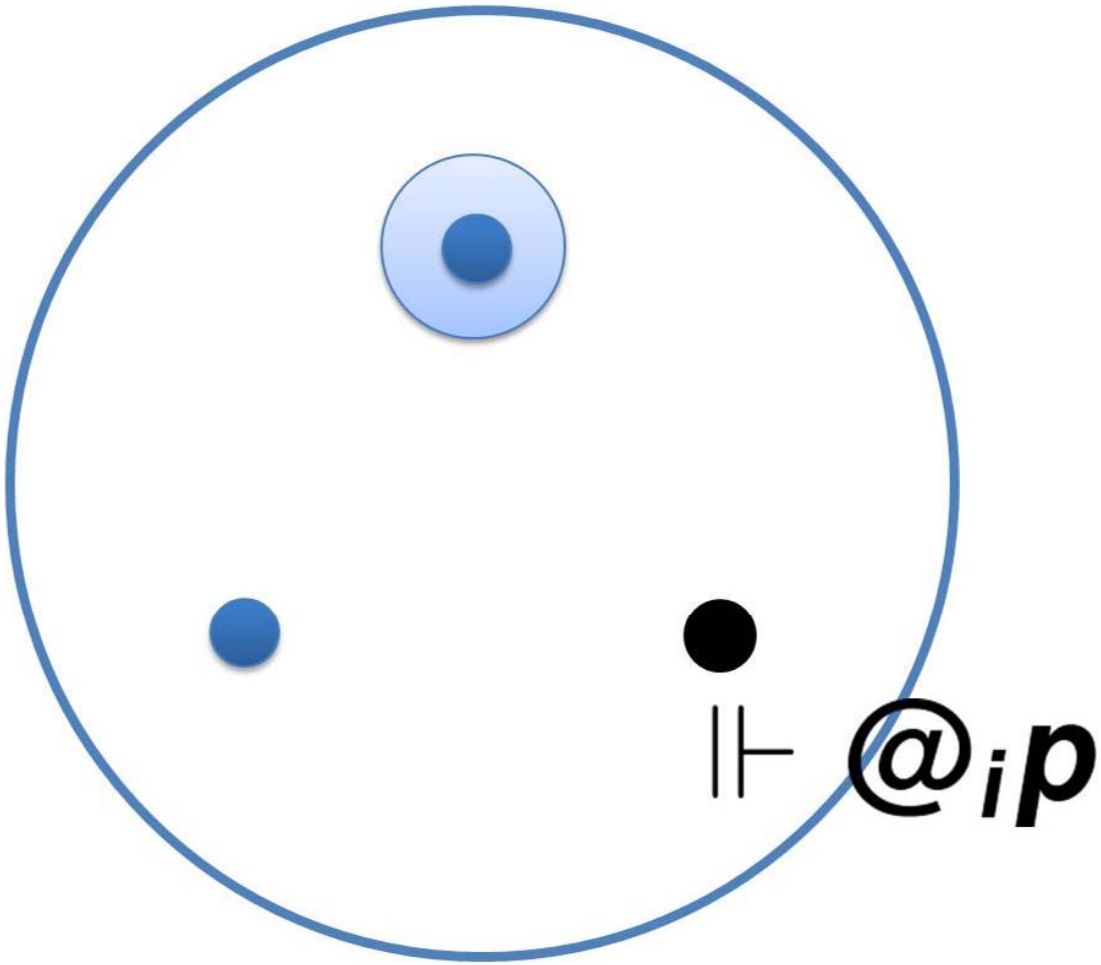

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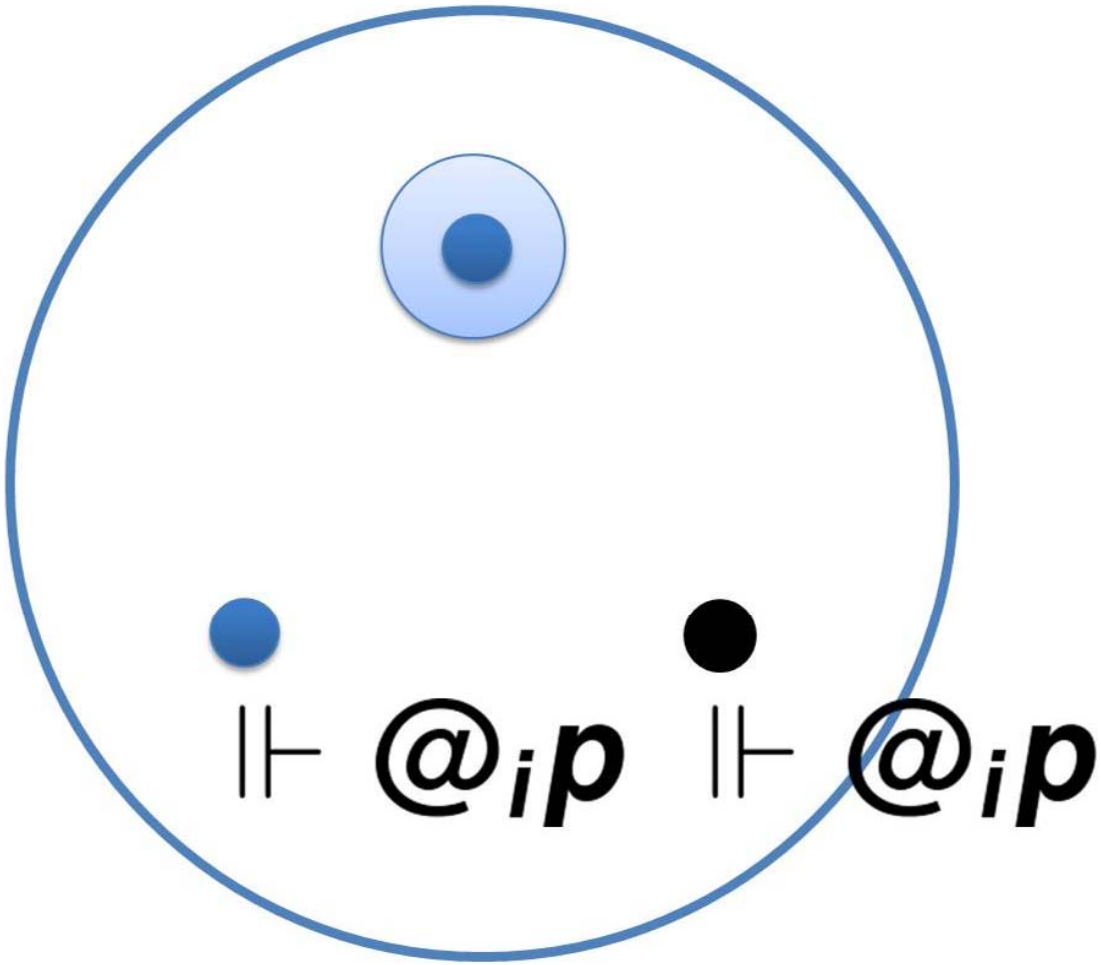

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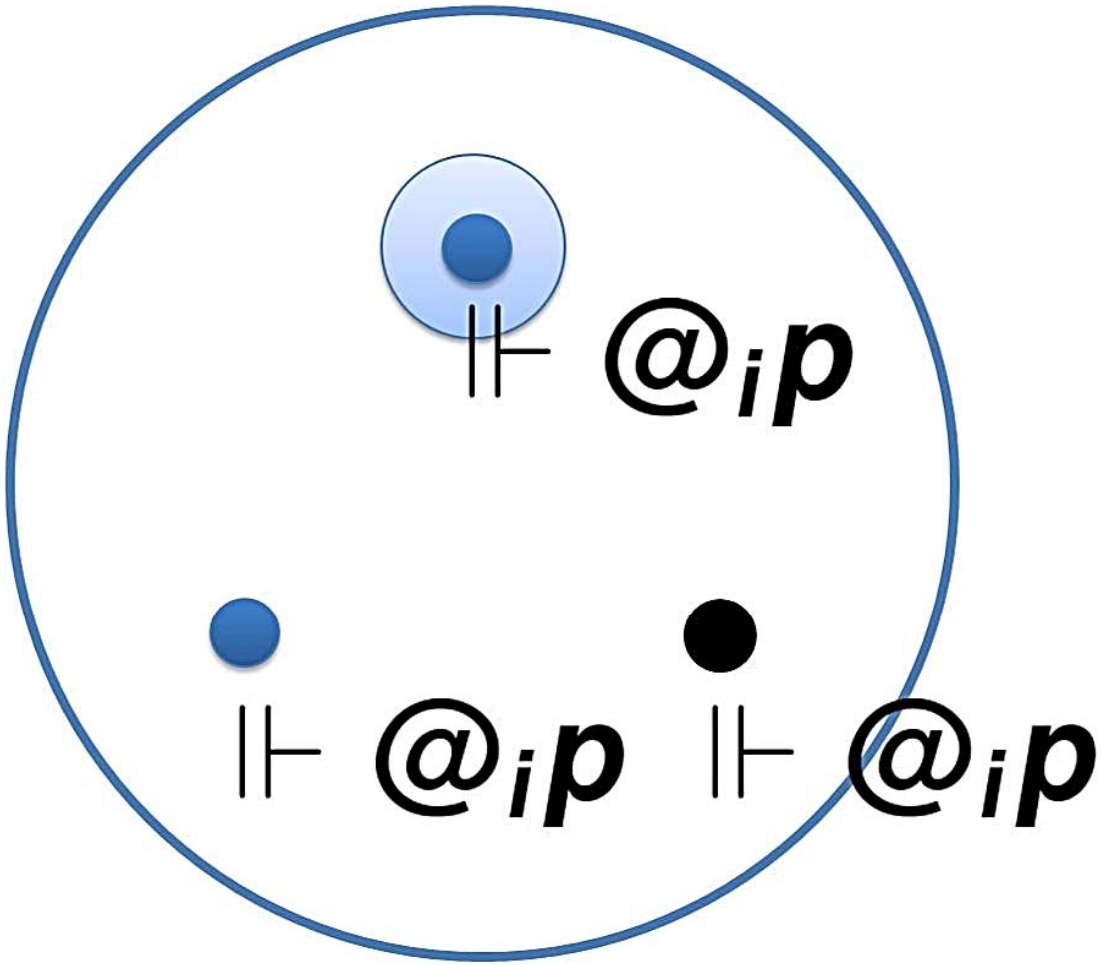

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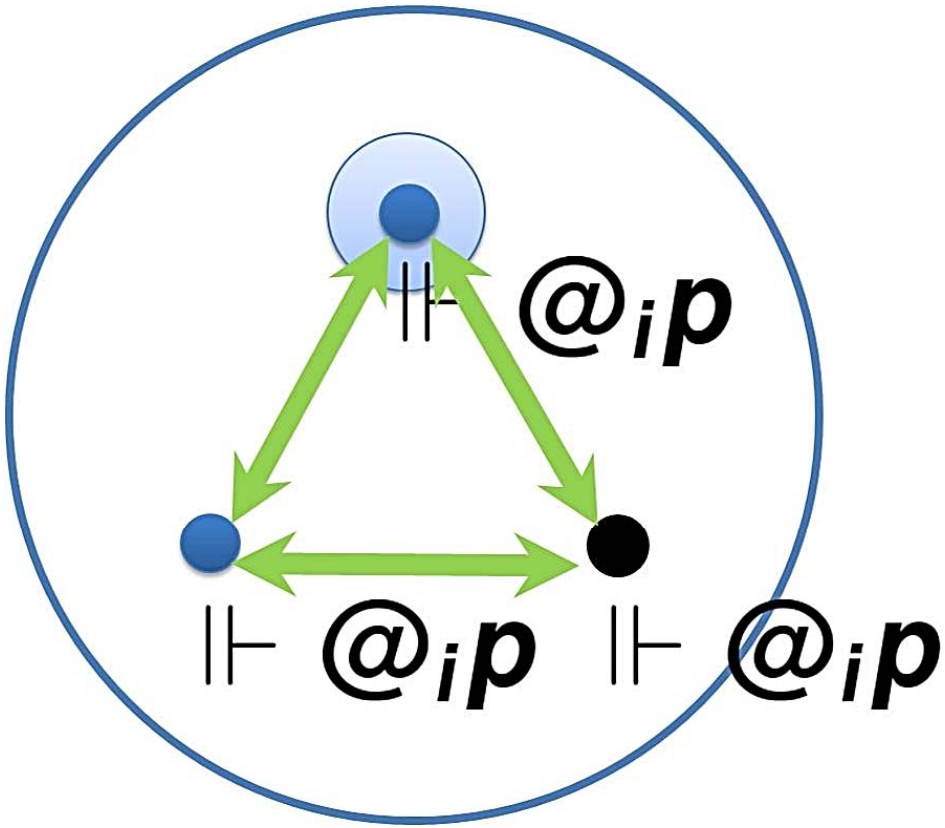
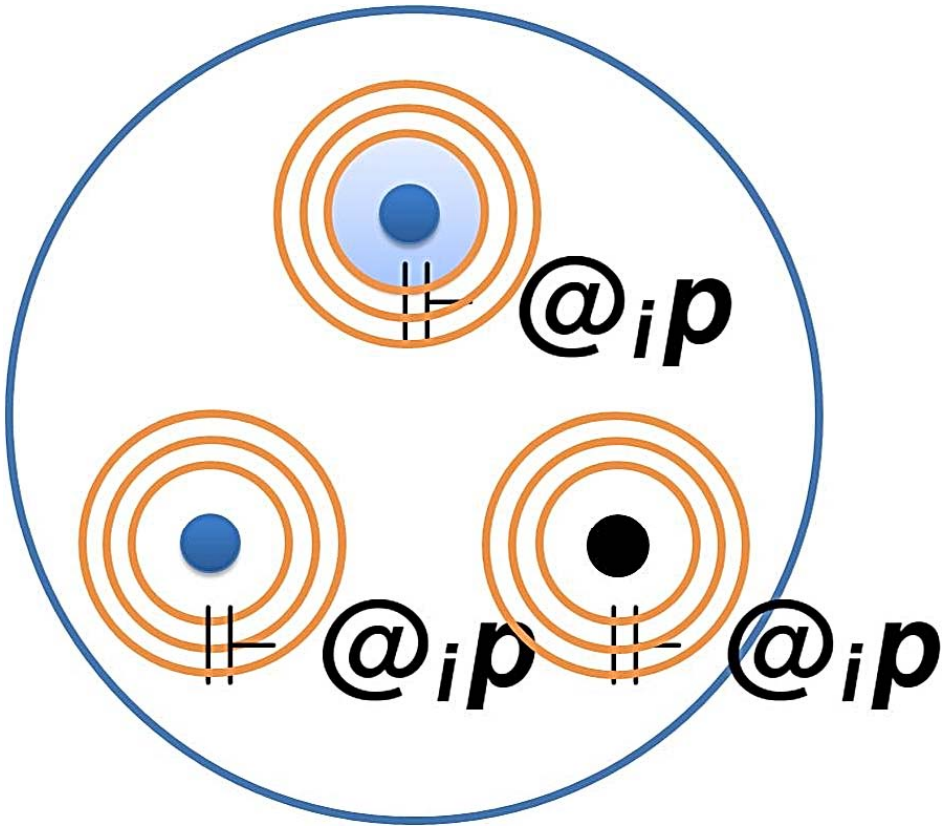


Structures on the domain are irrelevant to a hybridization.

Semantics for Nominals and Satisfaction Operators



Structures on the domain are irrelevant to a hybridization.

A Merit of Hybrid Logic

- 8th June is future.
- Katsuhiko drinks much on 8th June.
- Thus: Katsuhiko will drink much.

Within hybrid logic, we can prove the following as a theorem:

$$\langle \text{Future} \rangle i \wedge @_i p \rightarrow \langle \text{Future} \rangle p,$$

where i = '8th June' and p = 'Katsuhiko drinks much'.

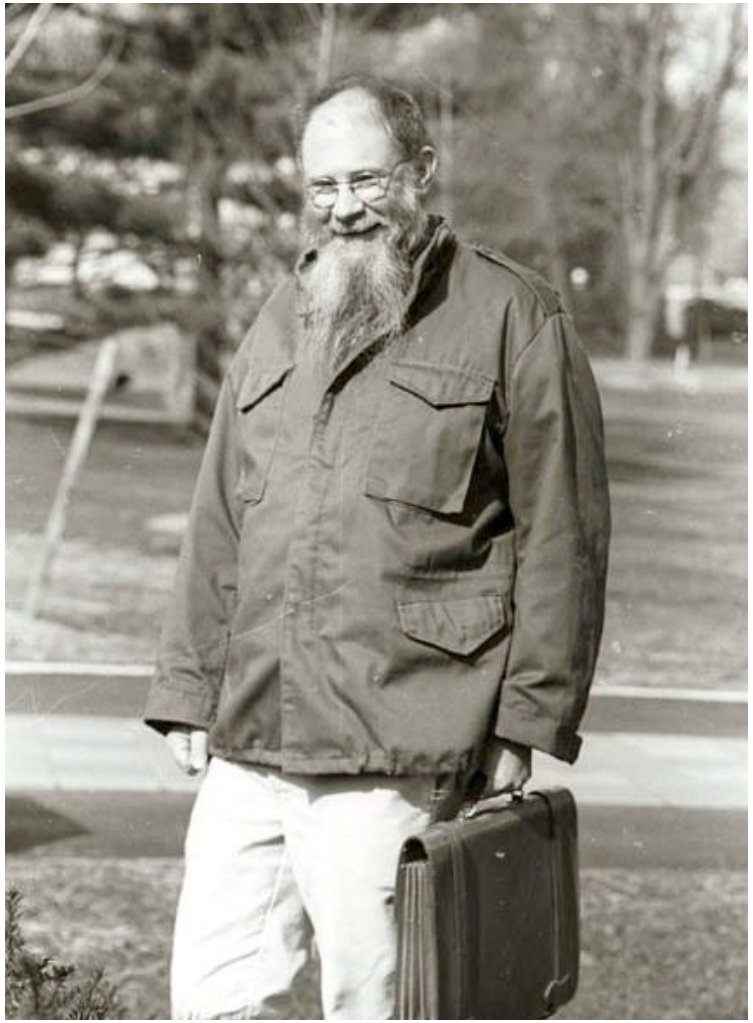
- Hybrid Logic enables us to formalize the inference containing both local & global information!

P. Blackburn

Arthur Prior and Hybrid Logic

Synthese, 150, pp.329-372, 2006.

David Lewis (1941 – 2001)



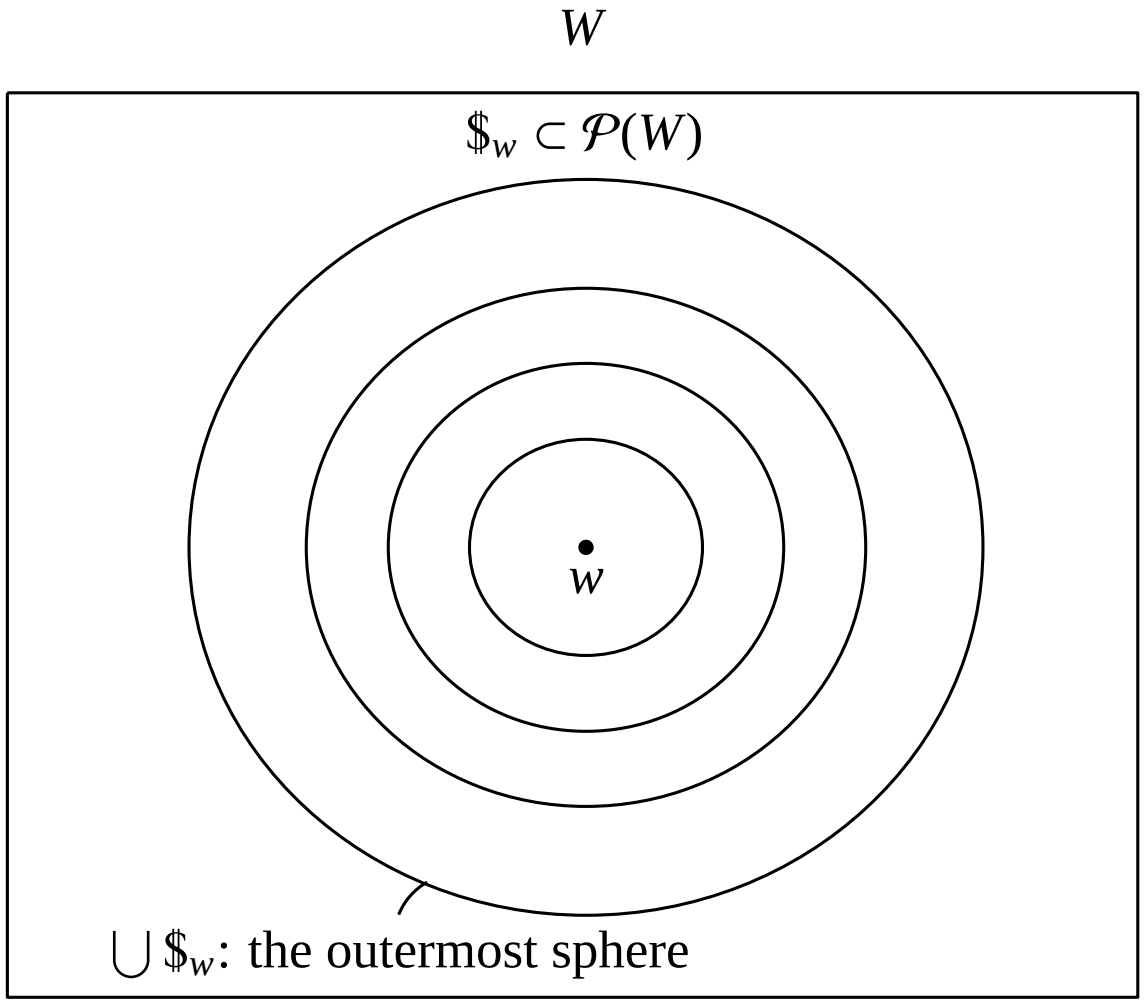
David Lewis' Analysis for Counterfactuals

- (Eg) If kangaroos had no tails, then they would topple over.
- 'If it were the case that φ, then it would be the case that ψ'.
- (kangaroos have no tails) $\Box\!\!\rightarrow$ (kangaroos topple over).
- David Lewis gives the truth condition of $\varphi \Box\!\!\rightarrow \psi$ at w as:

$(\varphi \wedge \psi)$-worlds are closer to w than $(\varphi \wedge \neg\psi)$-worlds.

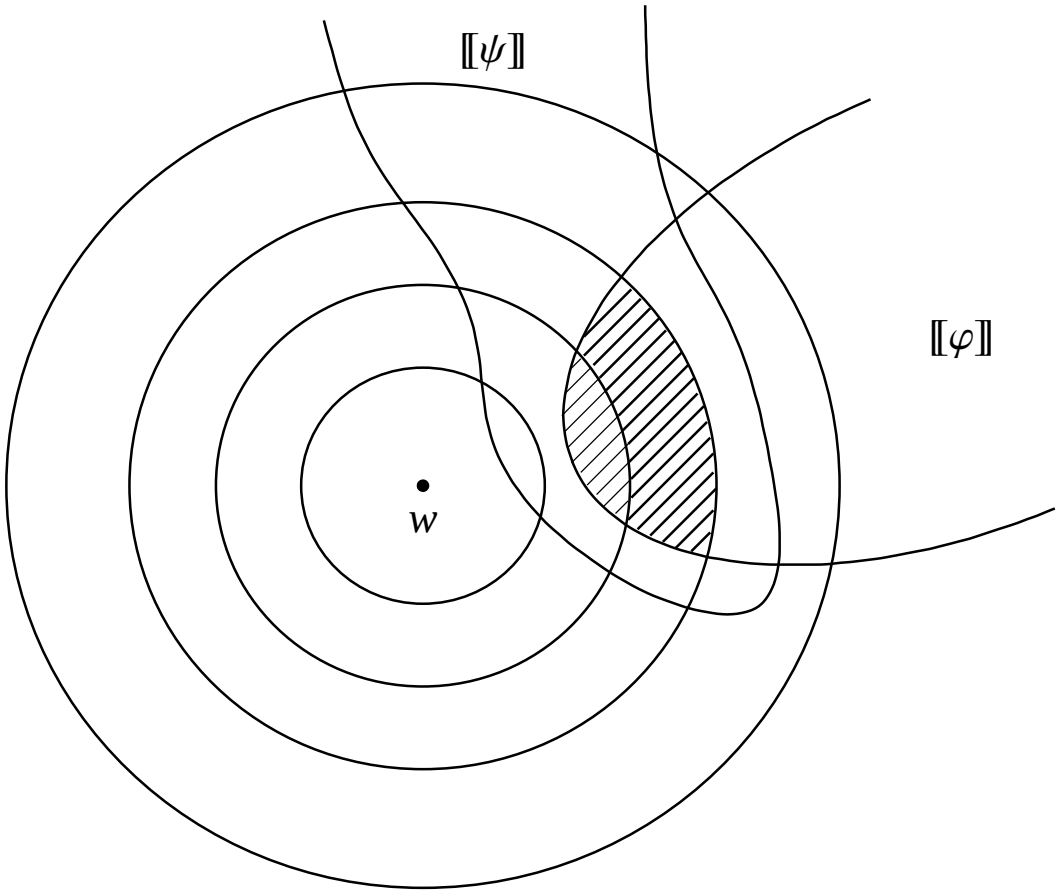
- How to define the notion of 'relative closeness'?

A System of Spheres



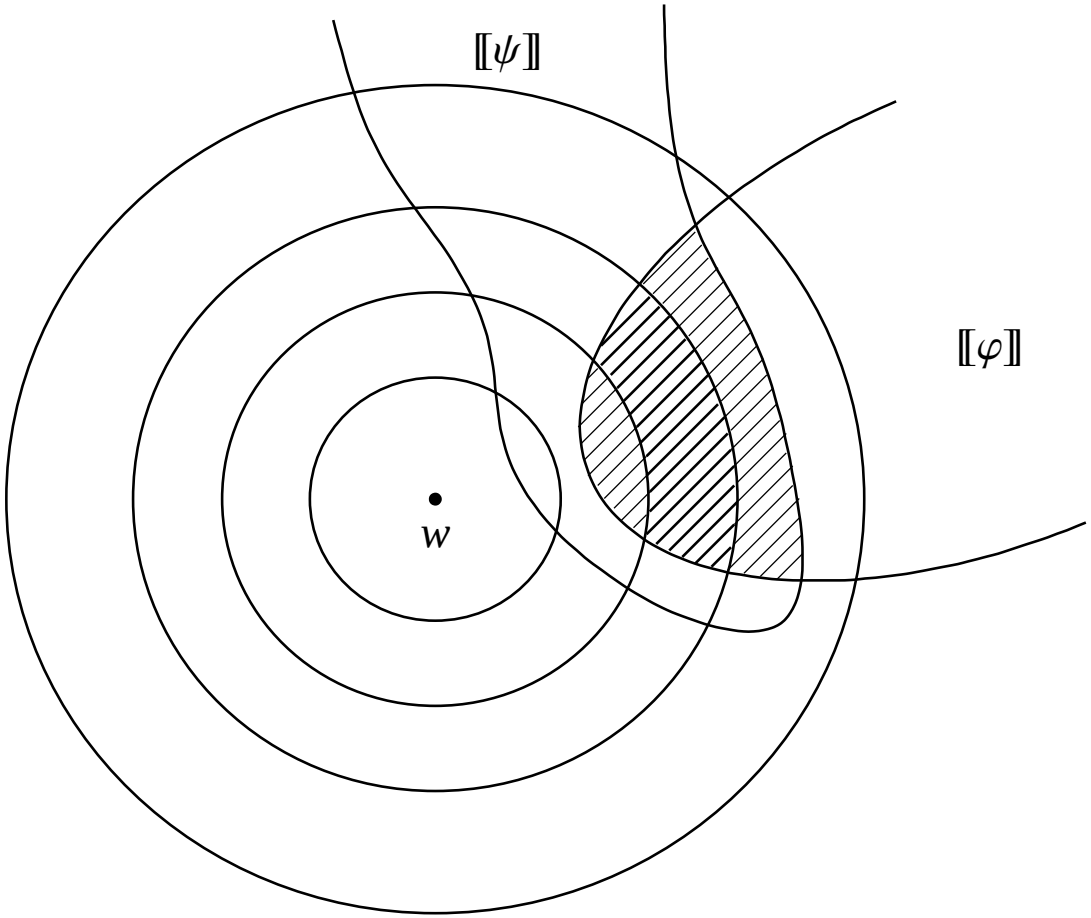
Truth Condition for Counterfactuals (1)

$\varphi \boxdot\!\!\rightarrow \psi$ is true at w iff:



Truth Condition for Counterfactuals (2)

$\varphi \boxdot\!\!\rightarrow \psi$ is true at w iff:



How to formalize?

- The pig is Mary.
- Mary is pregnant.
- Therefore: The pig is pregnant.

A Merit of Hybrid Logic

- 8th June is future.
- Katsuhiko drinks much on 8th June.
- Therefore: Katsuhiko will drink much.

'The' in the Ordinary Language

Lewis (1973, sec 5.3) considered Arthur Prior's 'egocentric' reading of proposition:

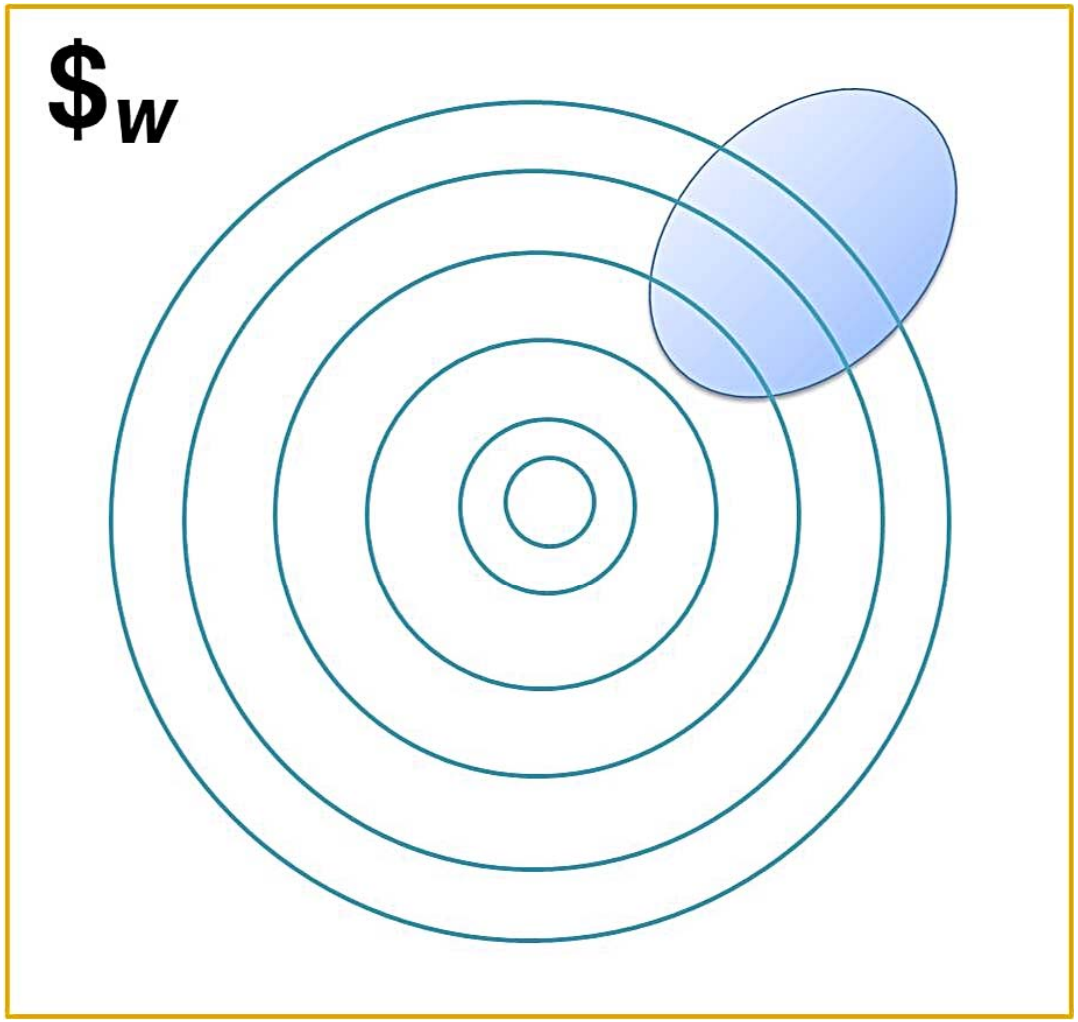
- p is true at w iff the individual w has the property p,

and proposed that:

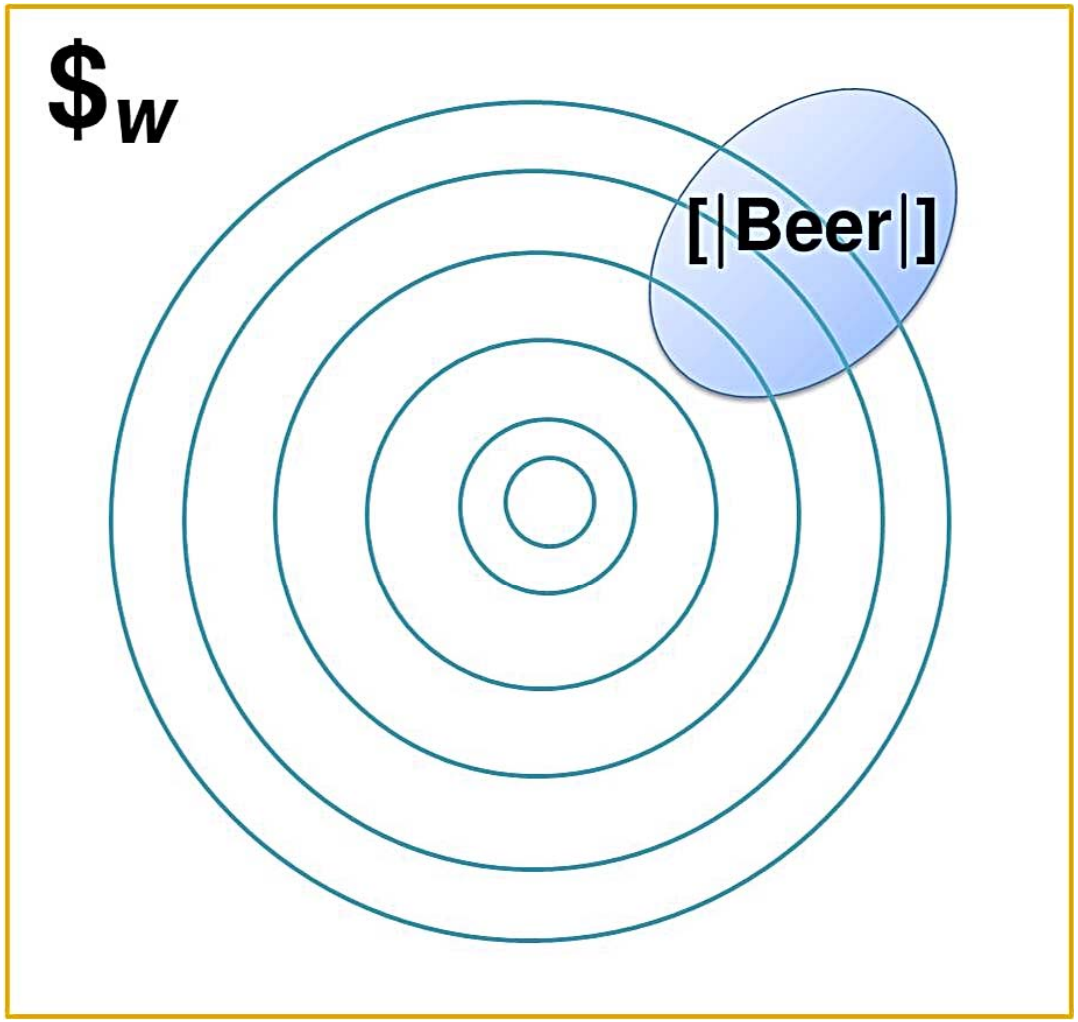
- $\varphi \mathrel{\Box\!\!\rightarrow} \psi$ expresses contextually definite descriptions: 'The x such that φ is such that ψ'.
- (Eg) 'The pig is grunting' (written: Pig $\mathrel{\Box\!\!\rightarrow}$ Grunting).

	Counterfactual Reading	Egocentric Reading
$w \in W$	worlds	things or individuals
$\$_w$	comparative similarity	comparative salience
$\varphi \mathrel{\Box\!\!\rightarrow} \psi$	counterfactual conditional	The x such that φ is such that ψ.

Comparative Salience



Comparative Salience



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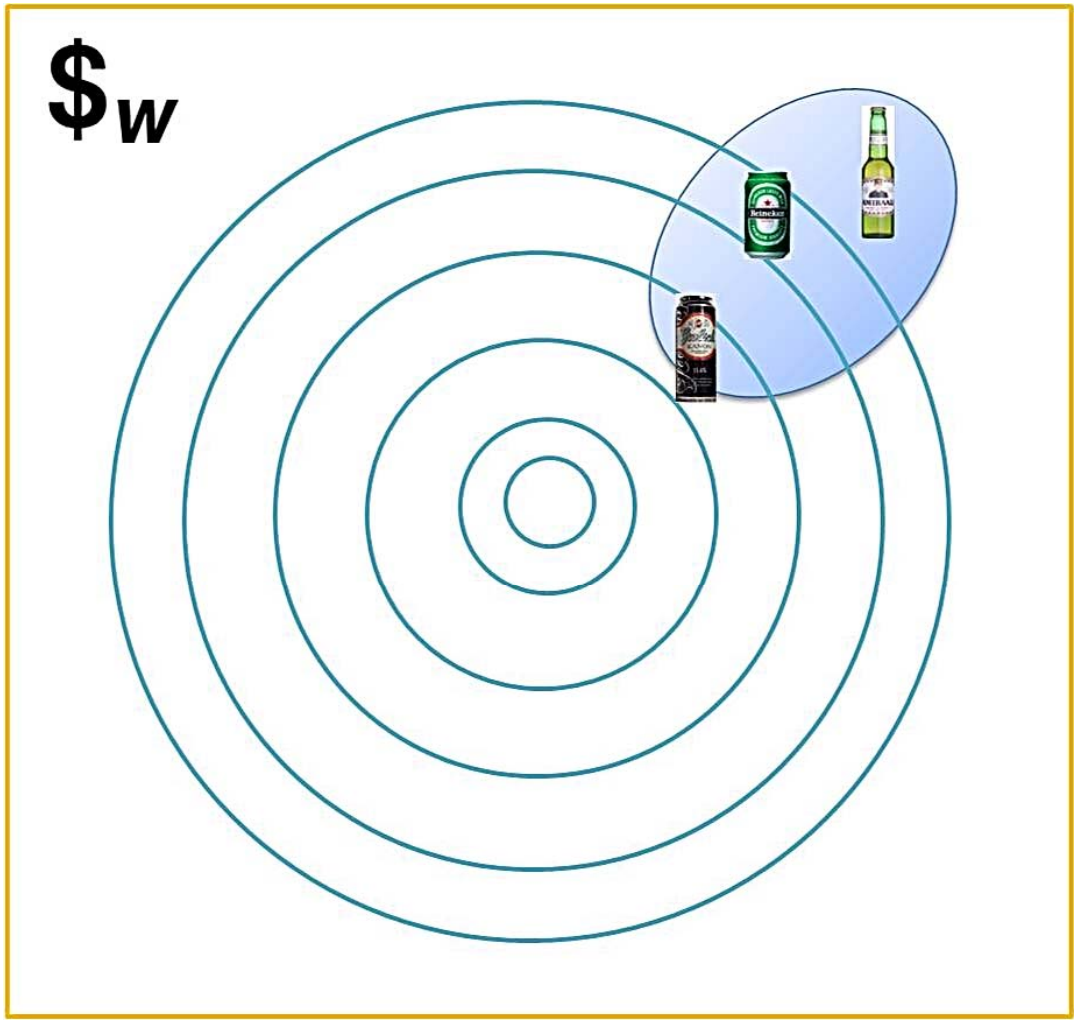
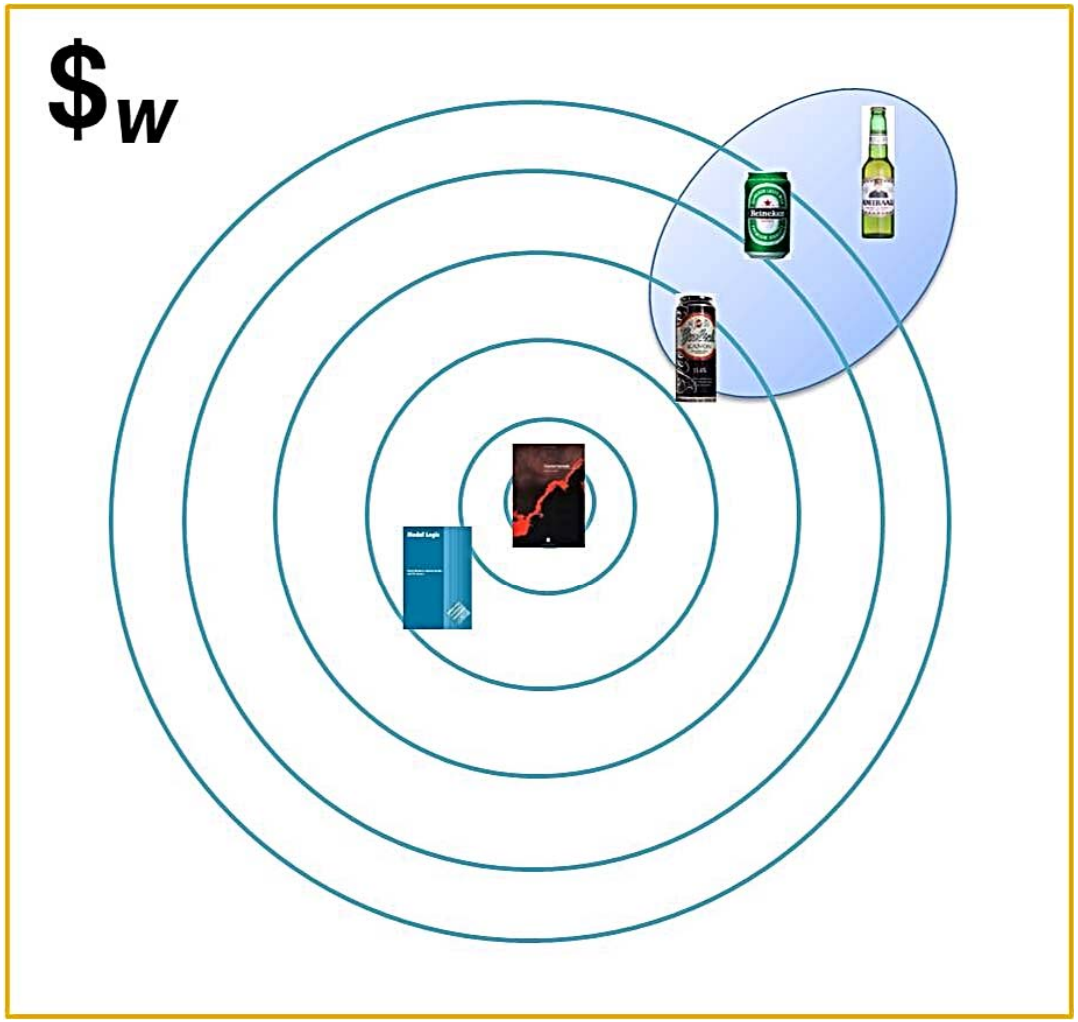
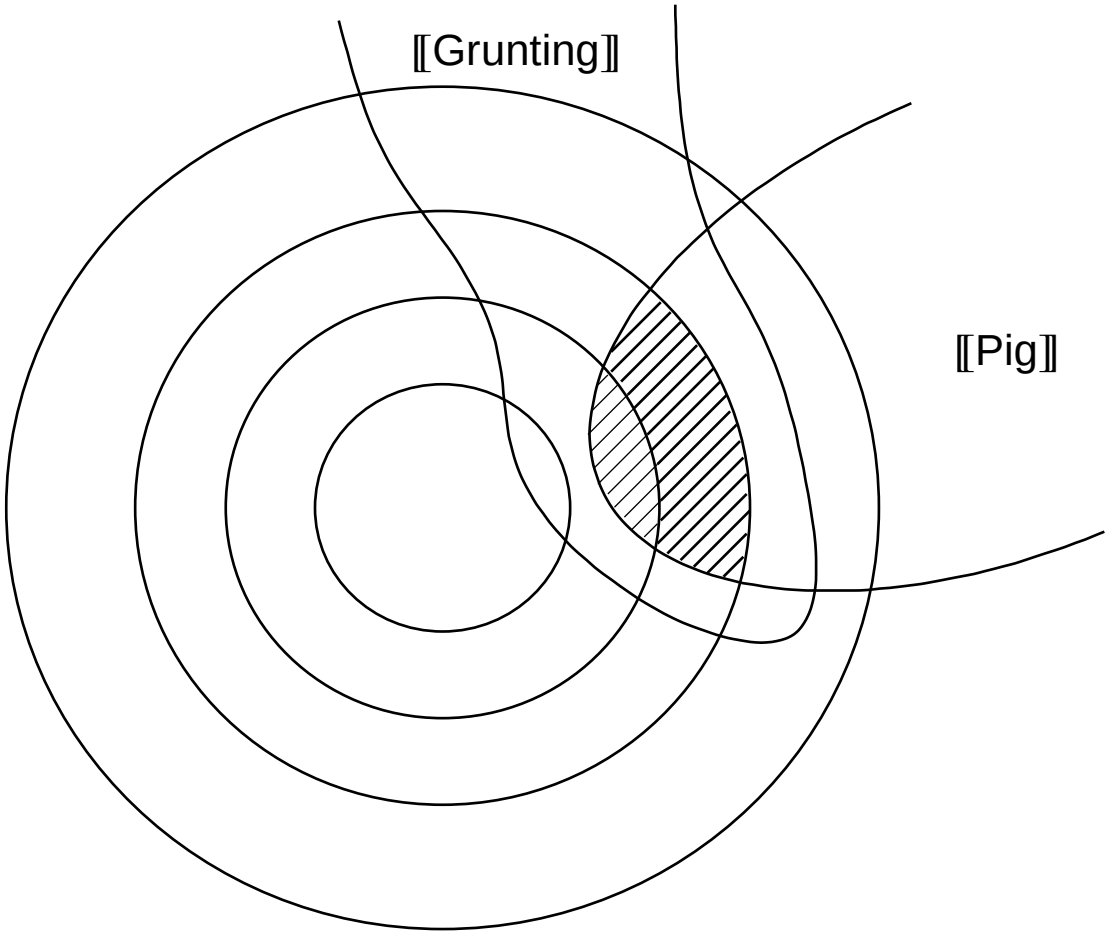


Comparative Salience



Contextually Definite Description

Pig $\Box\!\!\rightarrow$ Grunting is true at *w* iff:



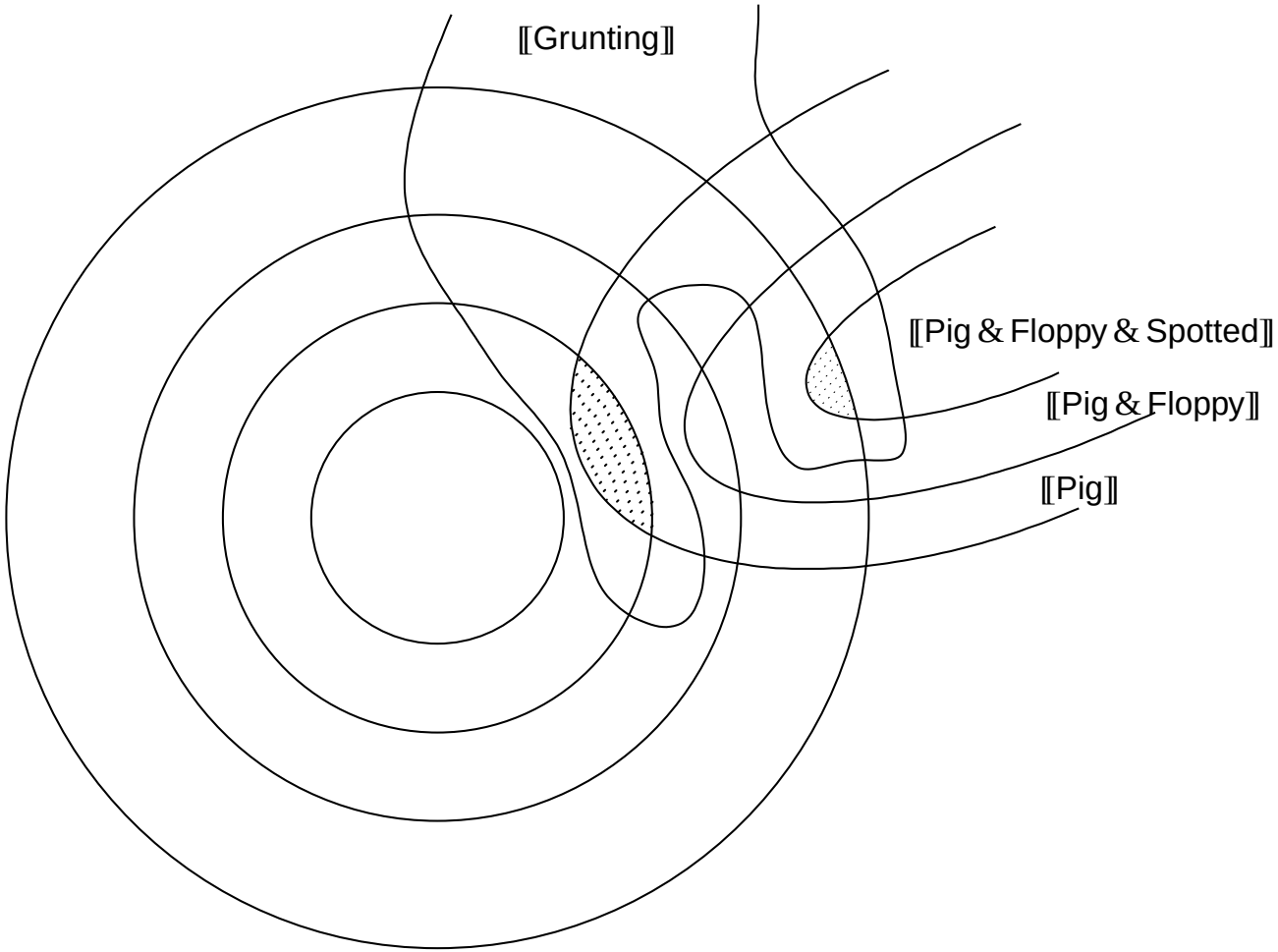
A Sequence of Egocentric Conditionals

1. The pig is grunting.
   - Pig $\Box\!\!\rightarrow$ Grunting.
2. The pig with floppy ears is not grunting.
   - (Pig $\wedge$ Floppy) $\Box\!\!\rightarrow$ $\neg$Grunting.
3. The spotted pig with floppy ears is grunting.
   - (Pig $\wedge$ Floppy $\wedge$ Spotted) $\Box\!\!\rightarrow$ Grunting.

ι-operator of FOL is not appropriate for this job, since we cannot make the following formulas true at the same time:

$$\text{Grunting}(\iota x \text{Pig}(x));$$
$$\neg\text{Grunting}(\iota x(\text{Pig}(x) \wedge \text{Floppy}(x))).$$

A System of Comparative Salience



How to formalize?

- **The pig is Mary.**
- **Mary is pregnant.**
- **Therefore: The pig is pregnant.**

A Need for Hybrid Formalism

To deal with 'Mary is pregnant', we need hybrid formalism.
Lewis also dealt with a similar kind of sentence:

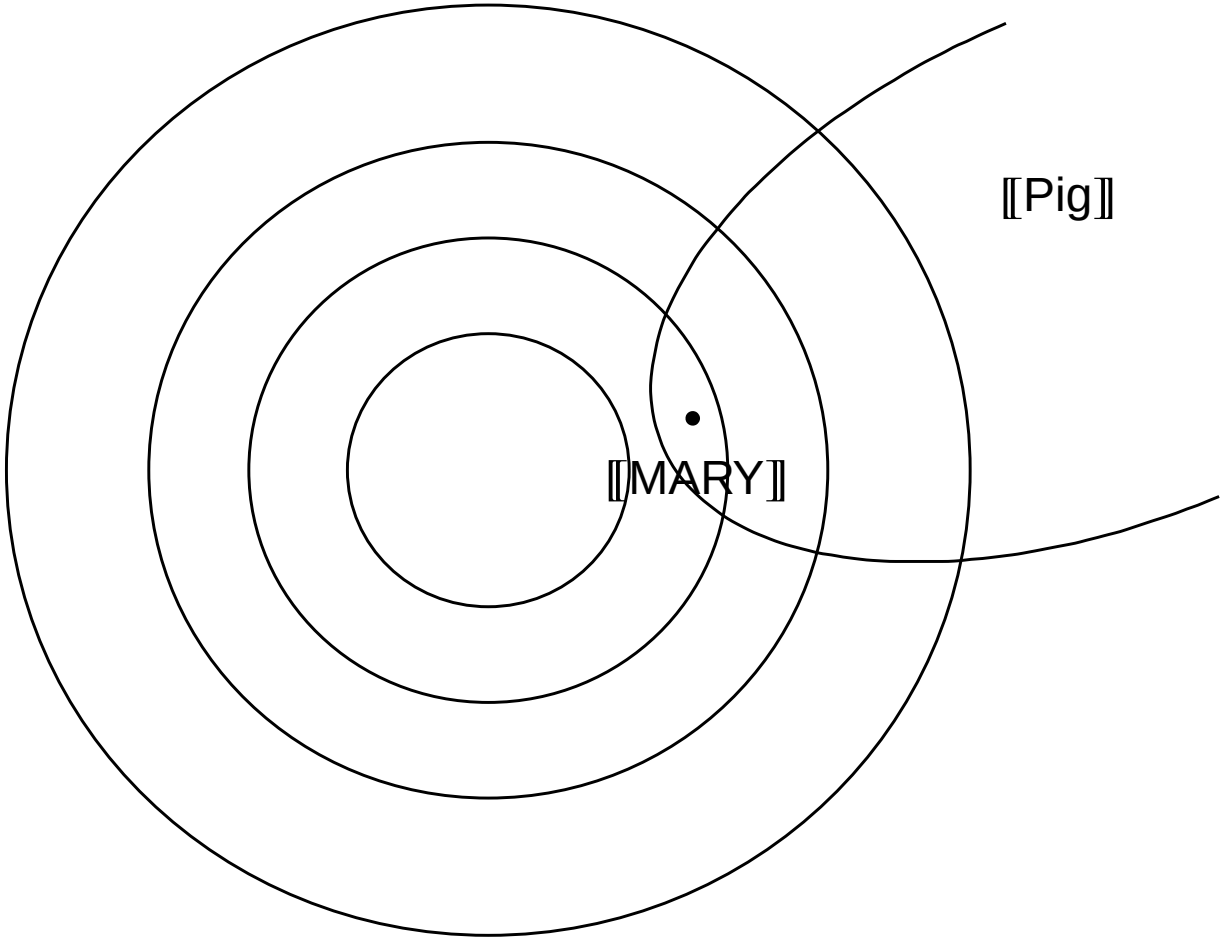
- "x is such that (the Anighito meteorite is an x such that x is a rock)" in Lewis (1973, p.112).

We write this as '$@_{\text{ANIGHITO METEORITE}}\text{Rock}$'.
If Lewis had known the modern development of hybrid logic, then he would hybridize his counterfactual logic!

Here David Lewis meets Arthur Prior again!

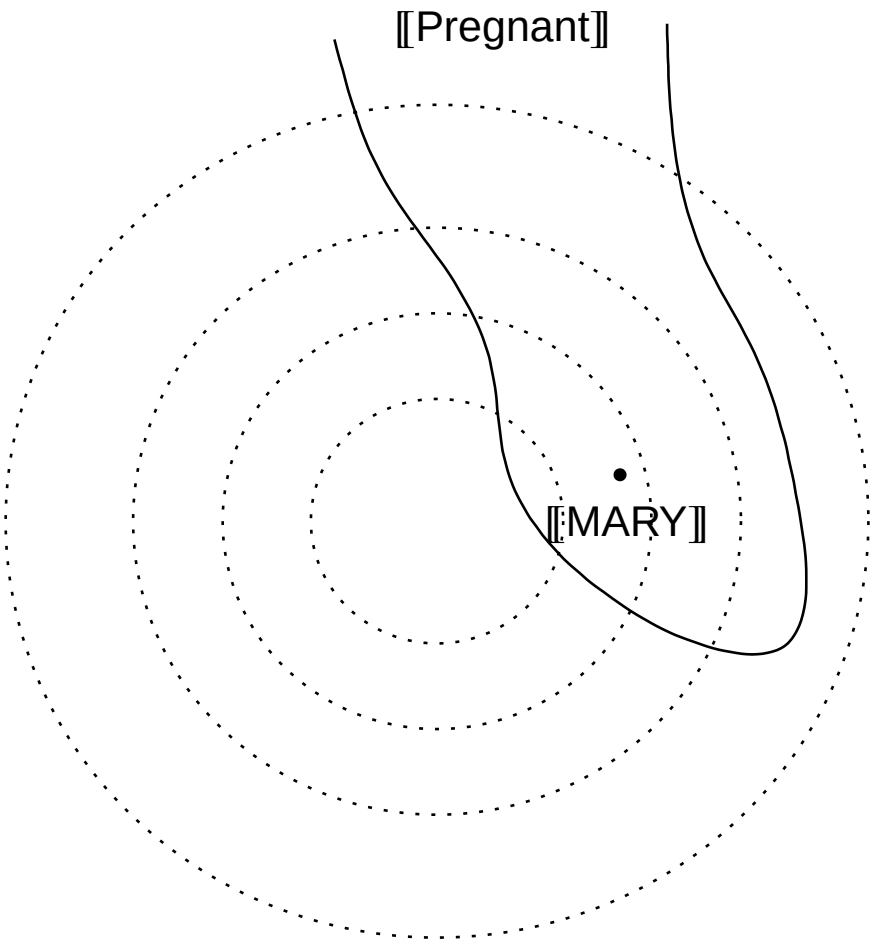
Semantic Understanding of the Motivating Example

The pig is Mary ($w \in [\![\text{Pig} \mathrel{\Box\!\!\rightarrow} \text{MARY}]\!]$).



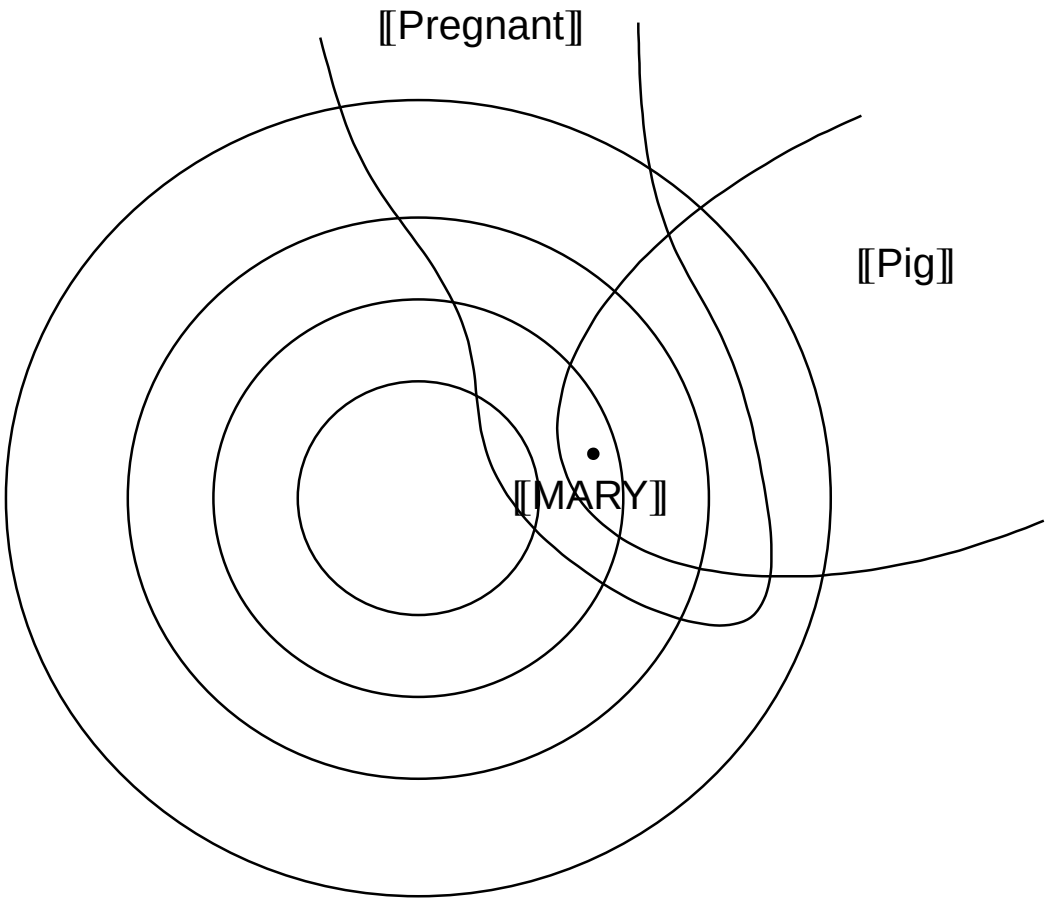
Semantic Understanding of the Motivating Example

Mary is Pregnant ($w \in [\![@_{\mathrm{MARY}}\mathrm{Pregnant}]\!]$).



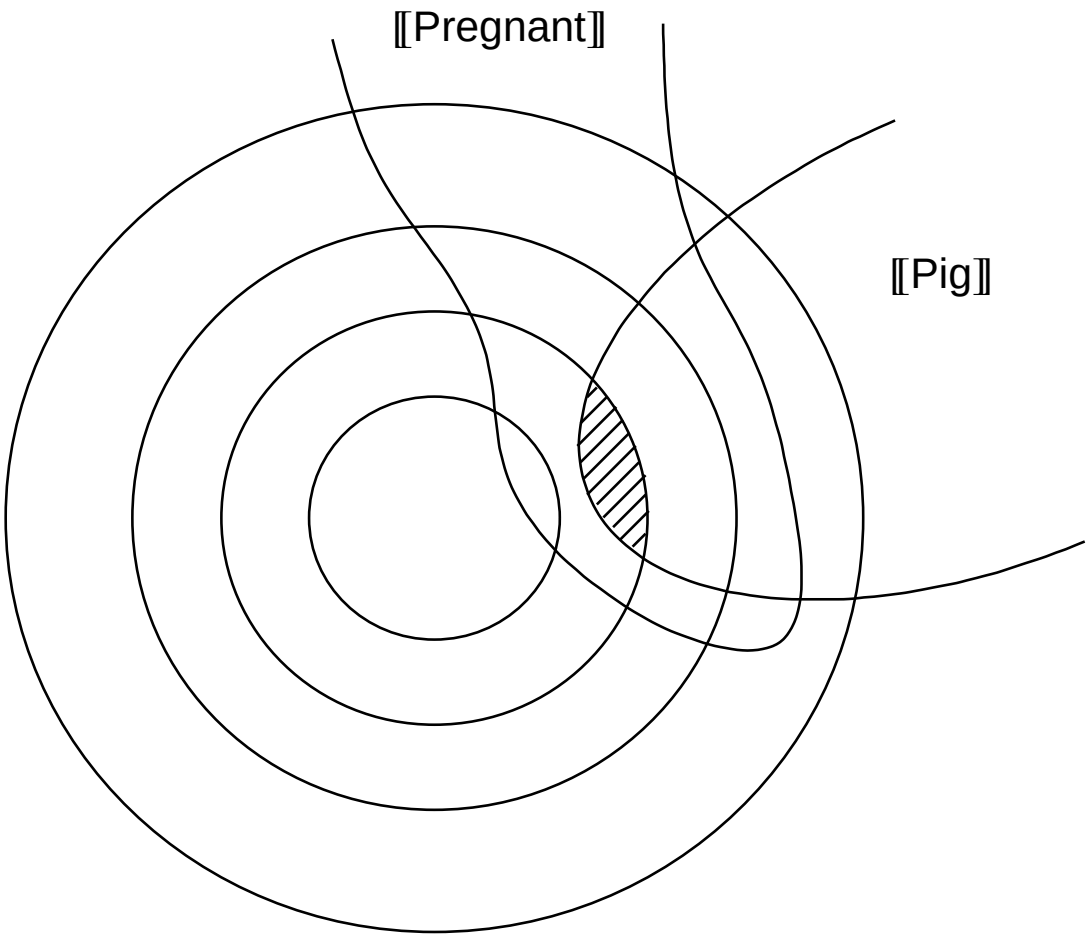
Semantic Understanding of the Motivating Example

$w \in [\![\text{Pig} \mathrel{\Box\!\!\rightarrow} \text{MARY}]\!]$ and $w \in [\![@_{\text{MARY}}\text{Pregnant}]\!]$.



Semantic Understanding of the Motivating Example

Therefore: The pig is pregnant ($w \in [\![\text{Pig} \boxright \text{Pregnant}]\!]$).



Fact (Lewis 1973)

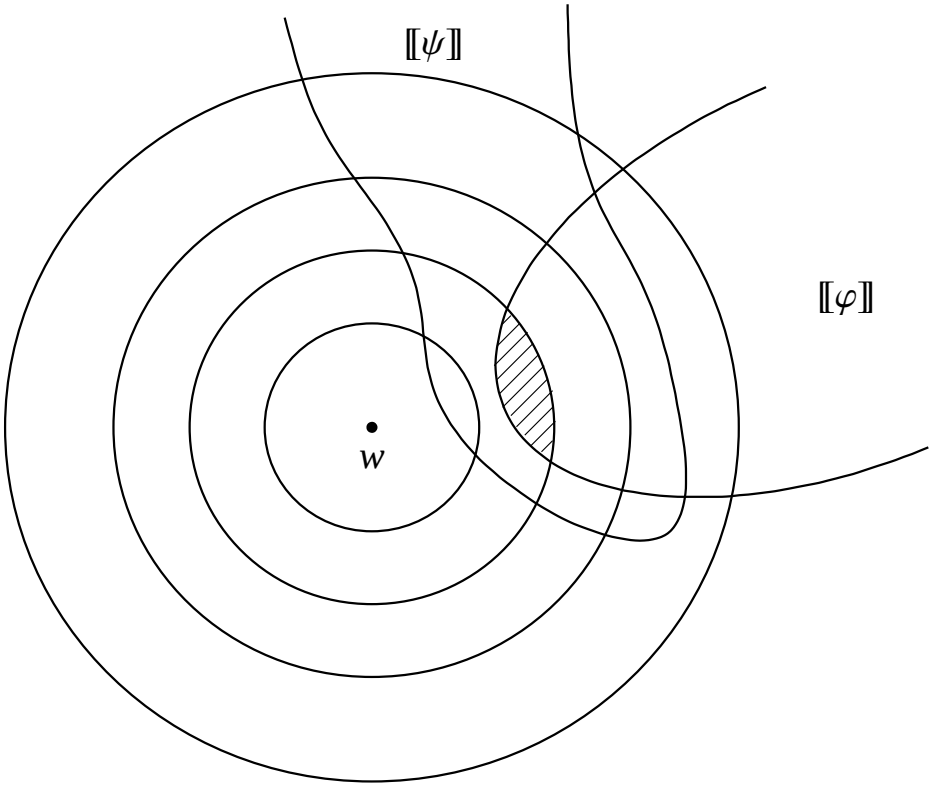
V is sound and complete wrt the class of finite sphere models. Thus, **V** is decidable.

Completeness and Decidability of $\mathbf{V}_{\mathcal{HC}(@)}$ (S. 2009)

$\mathbf{V}_{\mathcal{HC}(@)}$ is sound and complete wrt the class of finite sphere models. Thus, $\mathbf{V}_{\mathcal{HC}(@)}$ is decidable.

The Limit Assumption

- The Limit Assumption (LA) :
  there exist the most similar worlds wrt possible $[\![\varphi]\!]$.



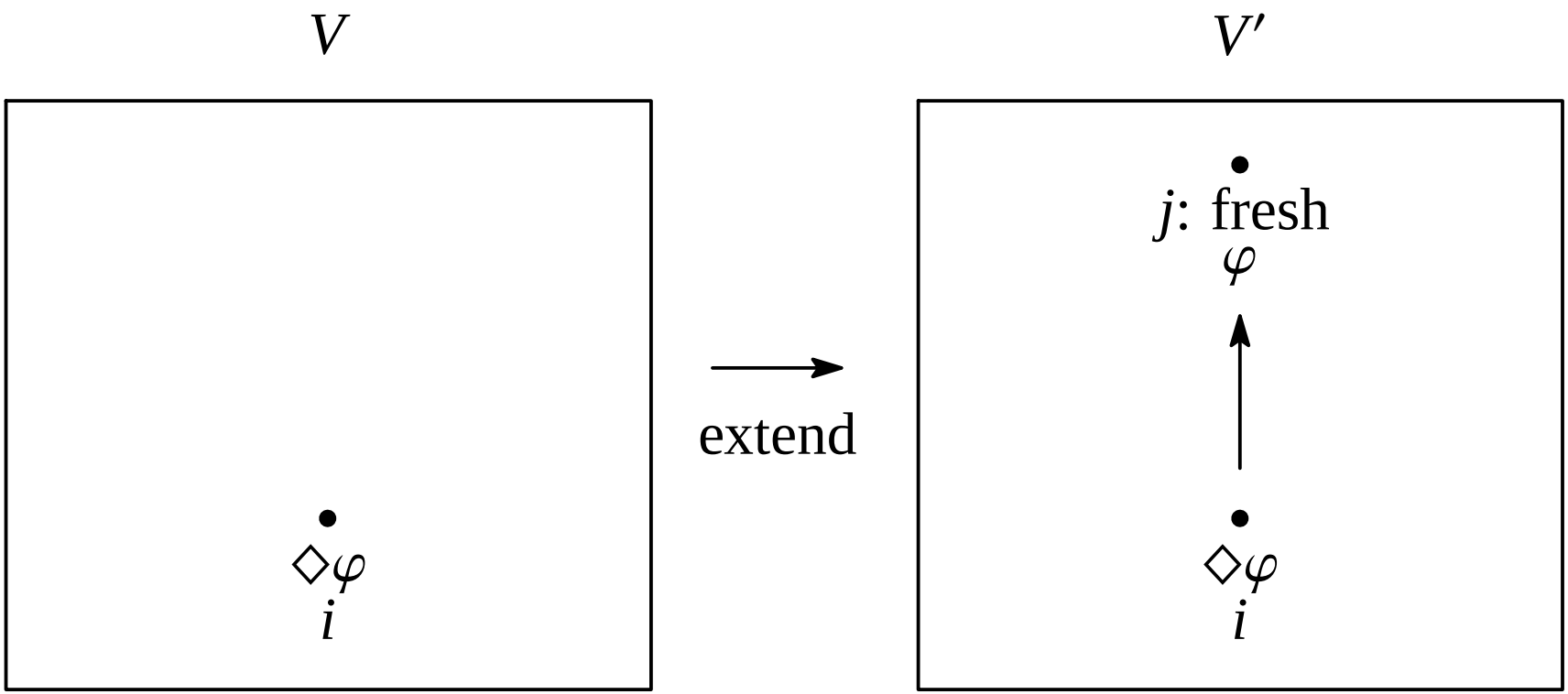
LA is Useful, But...

- LA gives us simpler semantics called selection function.
  - A multimodal Kripke frame with $\left\{ R_\varphi \mid \varphi \text{ is a formula} \right\}$.
- David Lewis disagrees with LA! It's because he can consider more and more similar φ-worlds.
- However, he states that there is no characteristic axiom for LA (Lewis (1973, p.121)).

Hybridization gives us an idea of characterization of LA!

Intuitive Meaning of **BG** on Kripke Frame

- **BG**: If $@_i \Diamond \varphi$ is consistent, then $@_i \Diamond j \wedge @_j \varphi$ is consistent for some fresh j.



Proof-rule characterizes the Limit Assumption

Let $\psi \diamond\!\!\rightarrow \varphi := \neg(\psi \mathrel{\Box\!\!\rightarrow} \neg\varphi)$ (Might Counterfactual).

BG : If $@_i \diamond \varphi$ is consistent, then $@_i \diamond j \wedge @_j \varphi$ is consistent for some fresh j.

CBG : If $@_i \psi \diamond\!\!\rightarrow \varphi$ is consistent, then $@_i \psi \diamond\!\!\rightarrow j \wedge @_j \varphi$ is consistent for some fresh j.

Let's say: A system of sphere admits **CBG** if every valuation satisfying $@_i \psi \diamond\!\!\rightarrow \varphi$ at some point can be extended to a valuation satisfying $@_i \psi \diamond\!\!\rightarrow j \wedge @_j \varphi$ (j: fresh) at some point.

CBG characterizes LA (S. 2009)

$\langle W, \$ \rangle$ satisfies LA iff $\langle W, \$ \rangle$ admits **CBG**.

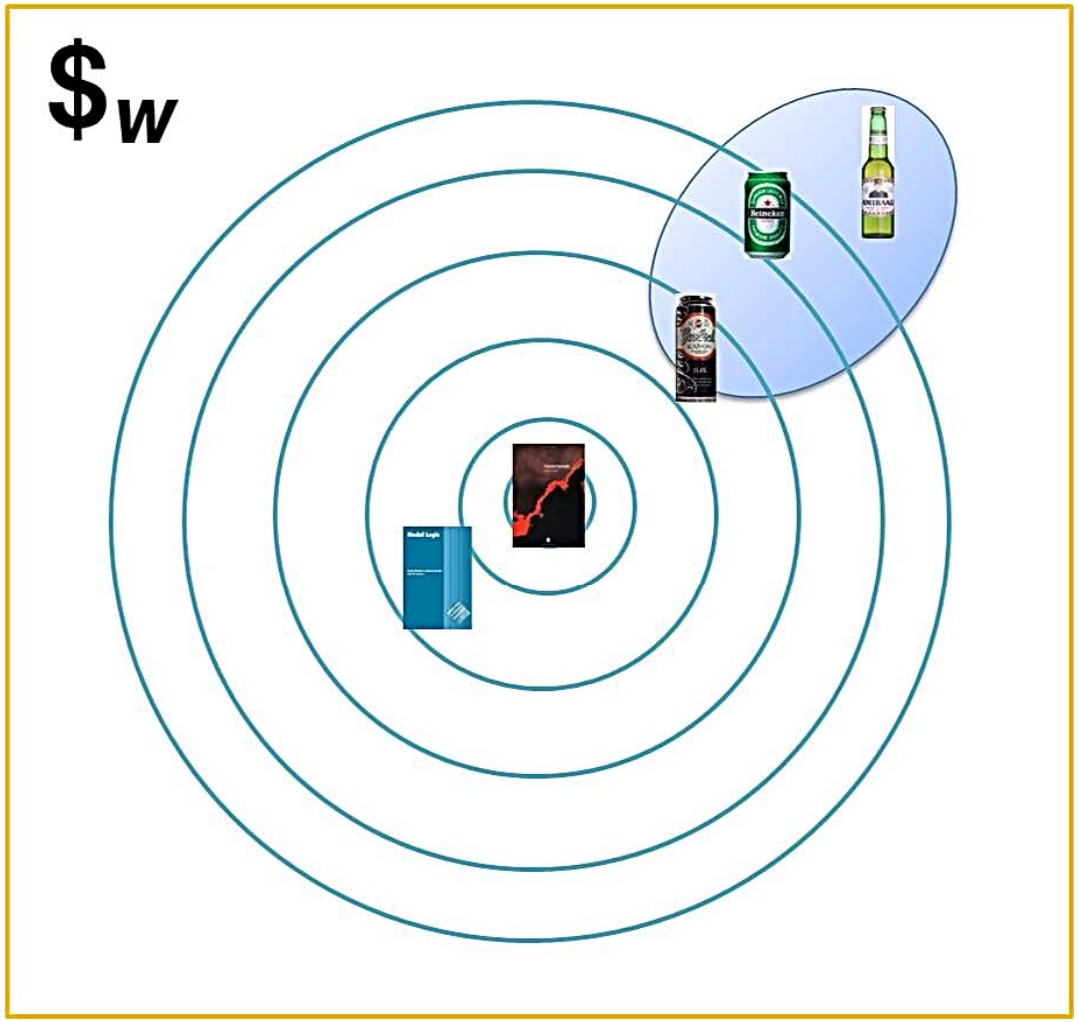
Further Directions: Comparative Salience Change

Lewis 1973, p.113

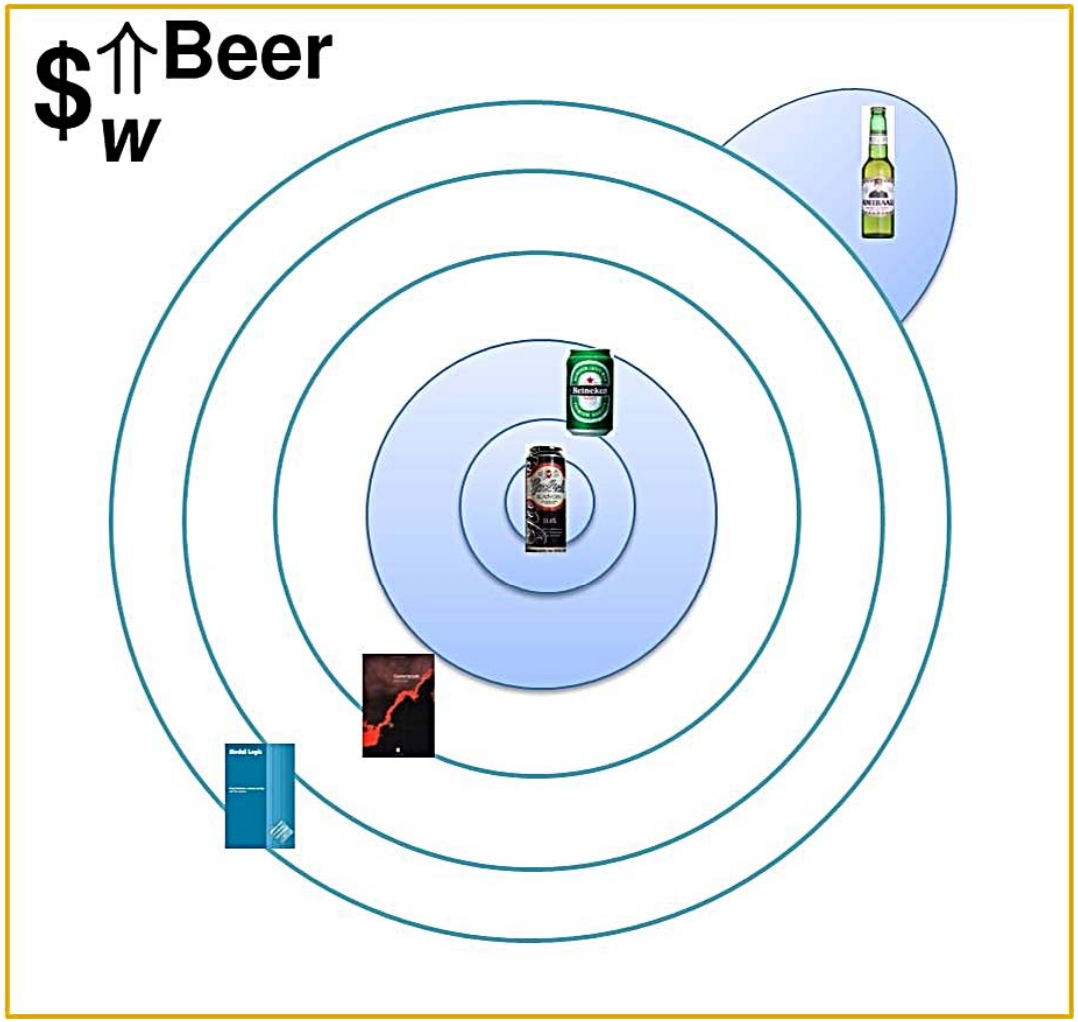
(I am speaking of how salient these things were *before* I started to think up examples of things that were not very salient; comparative salience is much shiftier even than comparative similarity.)

- Lewis did not explain how we can obtain a new comparative salience from old one in the formal setting.
- Radical Upgrade of DEL allows us to explicate this point.
- Dynamic Hybrid Counterfactual Logic

Radical Upgrade on Sphere Model: Idea



Radical Upgrade on Sphere Model: Idea



Take-home Message

Hybridization of Lewis' logic is a desirable step!

Thank You