[…]

(19*a*) **A farmer introduced a doctor to a priest. He told him that he already knew him.** Discourse

(*b*) $\{(f,d,p) \mid f$ is a farmer, d is a doctor, p is a priest$\}$ $=: R$

(*c*) $\{w,x,y,z)\} \mid w$ told x that y already knew $z\}$ $=: S$

(*d*) $R \oplus_{\{w=p,x=f,y=p,z=d\}} S = R \cap \{(f,d,p) \mid p$ told f that p already knew $d\}$

From Relations to Possibilities

(20*a*) $R \subseteq U^n = \underbrace{U \times \ldots \times U}_{n \text{ times}}$ set of n-tuples (Cartesian product)

(20*b*) $B^A = \{f \mid f: A \to B\}$ set of functions from A to B

(20*c*) $n = \{m \in \omega \mid m < n\} = \{0, \ldots, n\}$ von Neumann ordinals

(20*d*) $R \subseteq U^n = \{f: n \to U\}$ set of n-place sequences

(20*e*) $U^n \cong \{f: X \to U\} \Leftrightarrow |X| = n$ (extensional) possibilities

Discourse Representations

Basic symbols

predicates (with intrinsic -arities): $\boldsymbol{P}$, $\boldsymbol{Q}$, $\boldsymbol{R}$ …

Variables (a.k.a. *discourse referents*): $\boldsymbol{x}$, $\boldsymbol{y}$, $\boldsymbol{z}$,…

(binary) quantifiers: **∀**, **MOST**, **NO**,…

auxiliary symbols: **(**, **)**, **[**, **]**, **,**, **:**

Categories:

Main category: *DRS*

others: $Pred_n$, *Var*, *Cond*, *Quant*

(Proper) Expressions (and their free variables)

(i) $\boldsymbol{R} \in Pred_n, \boldsymbol{x_1}, \ldots, \boldsymbol{x_n} \in Var \Rightarrow \boldsymbol{R(x_1, \ldots, x_n)} \in Cond$;
$Fr(\boldsymbol{R(x_1, \ldots, x_n)}) = \{\boldsymbol{x_1}, \ldots, \boldsymbol{x_n}\}$

(ii) $\varphi, \psi \in Cond \Rightarrow [\varphi.\psi] \in Cond$;
$Fr([\varphi.\psi]) = Fr(\varphi) \cup Fr(\psi)$

(iii) $\boldsymbol{X} \subseteq Var, \varphi \in Cond \Rightarrow [\boldsymbol{X}\text{:}\ \varphi] \in DRS$;
$Fr([\boldsymbol{X}\text{:}\ \varphi]) = Fr(\varphi) \setminus \boldsymbol{X}$

(iv) $\boldsymbol{Q} \in Quant, \boldsymbol{K}, \boldsymbol{K'} \in DRS, \boldsymbol{x} \in Fr(\boldsymbol{K}) \cap Fr(\boldsymbol{K'}) \Rightarrow [\boldsymbol{K} \langle \boldsymbol{Qx} \rangle \boldsymbol{K'}] \in Cond$;
$Fr([\boldsymbol{K} \langle \boldsymbol{Qx} \rangle \boldsymbol{K'}]) = (Fr(\boldsymbol{K}) \cup Fr(\boldsymbol{K'})) \setminus \{\boldsymbol{x}\}$

Fairly standard interpretation

Models: $\mathscr{M} = (\mathscr{U}_{\mathscr{M}}, \mathscr{F}_{\mathscr{M}})$ where: $\mathscr{U}_{\mathscr{M}} \neq \emptyset$, $\mathscr{F}_{\mathscr{M}}(\boldsymbol{R}) \subseteq \mathscr{U}_{\mathscr{M}}{}^n$ and
$\mathscr{F}_{\mathscr{M}}(\boldsymbol{Q}) \subseteq \wp(\mathscr{U}_{\mathscr{M}}) \times \wp(\mathscr{U}_{\mathscr{M}})$ whenever $\boldsymbol{R} \in Pred_n$ and $\boldsymbol{Q} \in Quant$

Variable assignments g (based on model $\mathscr{M}$): $Var \rightsquigarrow \mathscr{U}_{\mathscr{M}}$ (partial functions)

Value $[\![\boldsymbol{A}]\!]^g$ of expression $\boldsymbol{A}$ *given* $\mathscr{M}$ *and (suitable*)* g:

(i') $[\![\boldsymbol{R}(\boldsymbol{x}_1,\ldots,\boldsymbol{x}_n)]\!]^{\mathscr{M},g} = \begin{cases} 1 \text{ iff } (g(\boldsymbol{x}_1),\ldots,g(\boldsymbol{x}_n)) \in \mathscr{F}_{\mathscr{M}}(\boldsymbol{R}) \\ 0 \text{ iff } (g(\boldsymbol{x}_1),\ldots,g(\boldsymbol{x}_n)) \notin \mathscr{F}_{\mathscr{M}}(\boldsymbol{R}) \end{cases}$

(i*) where $\{\boldsymbol{x}_1,\ldots,\boldsymbol{x}_n\} \subseteq dom(g)$

(ii') $[\![[\varphi.\psi]]\!]^{\mathscr{M},g} = [\![\varphi]\!]^{\mathscr{M},g} \times [\![\psi]\!]^{\mathscr{M},g}$

(ii*) where $[\![\varphi]\!]^{\mathscr{M},g}$ and $[\![\psi]\!]^{\mathscr{M},g}$ are defined

(iii') $[\![[\boldsymbol{X}\text{: }\varphi]]\!]^{\mathscr{M},g} = \{f\text{: }\boldsymbol{X} \to \mathscr{U}_{\mathscr{M}} \mid [\![\varphi]\!]^{\mathscr{M},g\cup f} = 1\}$

(iii*) where $dom(g) \cap \boldsymbol{X} = \emptyset$ and $[\![\varphi]\!]^{\mathscr{M},g\cup f}$ is defined

(iv') $[\![[\boldsymbol{K} <\boldsymbol{Qx}> \boldsymbol{K'}]]\!]^{\mathscr{M},g} = 1$

iff $(\{u\in\mathscr{U}_{\mathscr{M}} \mid (\exists f)\ f\cup\{(\boldsymbol{x},u)\} \in [\![\boldsymbol{K}]\!]^{\mathscr{M},g}\},$

$\{u\in\mathscr{U}_{\mathscr{M}} \mid (\exists f,h)\ [f\cup\{(\boldsymbol{x},u)\} \in [\![\boldsymbol{K}]\!]^{\mathscr{M},g}\ \&\ h\in[\![\boldsymbol{K'}]\!]^{\mathscr{M},g\cup f\cup\{(\boldsymbol{x},u)\}}]\}) \in \mathscr{F}_{\mathscr{M}}(\boldsymbol{Q})$

(iv*) where $[\![\boldsymbol{K}]\!]^{\mathfrak{M},g}$ and $[\![\boldsymbol{K'}]\!]^{\mathfrak{M},g\cup f\cup\{(\boldsymbol{x},u)\}}$ are defined for all $f\in[\![\boldsymbol{K}]\!]^{\mathfrak{M},g}$ and $u\in\mathscr{U}_{\mathscr{M}}$

<u>Highly non-standard interpretation</u>

Denotation $\|\boldsymbol{A}\|^g$ of expression $\boldsymbol{A}$ *given* model $\mathscr{M}$ (as above) *and* g: $Fr(\boldsymbol{A}) \to \mathscr{U}_{\mathscr{M}}$

Notational convention: $\|A\|^{\mathscr{M},\bar{g}} = \|A\|^{\mathscr{M},g\restriction Fr(\boldsymbol{A})}$

(i'') $\|\boldsymbol{R}(\boldsymbol{x}_1,\ldots,\boldsymbol{x}_n)\|^{\mathscr{M},g} = \begin{cases} 1 \text{ iff } (g(\boldsymbol{x}_1),\ldots,g(\boldsymbol{x}_n)) \in \mathscr{F}_{\mathscr{M}}(\boldsymbol{R}) \\ 0 \text{ iff } (g(\boldsymbol{x}_1),\ldots,g(\boldsymbol{x}_n)) \notin \mathscr{F}_{\mathscr{M}}(\boldsymbol{R}) \end{cases}$ [≈ **(i')**]

(ii'') $\|[\varphi.\psi]\|^{\mathscr{M},g} = \|\varphi\|^{\mathscr{M},\bar{g}} \times \|\psi\|^{\mathscr{M},\bar{g}}$ [≈ **(ii')**]

(iii'') $\|[\boldsymbol{X}\text{: }\varphi]\|^{\mathscr{M},g} = \blacklozenge\{g\text{: }\boldsymbol{X} \to U_{\mathscr{M}} \mid \|\varphi\|^{\mathscr{M},f\cup g} = 1\}$

where $\blacklozenge R = \{h\circ f \mid h\in R\ \&\ f\text{: } |dom(h)| \xrightarrow[\text{onto}]{\text{1-1}} dom(h)\}$

(iv'') $\|[\boldsymbol{K} <\boldsymbol{Qx}> \boldsymbol{K'}]\|^{\mathscr{M},g} = 1$

iff $(\{u\in\mathscr{U}_{\mathscr{M}} \mid (\exists f)\ f[^{\boldsymbol{x}}/_u] \in \bigcup\|\boldsymbol{K}\|^{\mathscr{M},\bar{g}}\},$ [≈ **(iv')**]

$\{u\in\mathscr{U}_{\mathscr{M}} \mid (\exists h)\ [h[^{\boldsymbol{x}}/_u] \in \bigcup\|\boldsymbol{K} \oplus \boldsymbol{K'}\|^{\mathscr{M},\bar{g}}]\}) \in \mathscr{F}_{\mathscr{M}}(\boldsymbol{Q})$ non-compositional

where $[\boldsymbol{X}\text{: }\varphi] \oplus [\boldsymbol{Y}\text{: }\psi] = [\boldsymbol{X}\cup\boldsymbol{Y}\text{: }[\varphi.\psi]]$ *Merge*

and $g[^{\boldsymbol{x}}/_u] = (g \setminus \{(\boldsymbol{x},g(\boldsymbol{x}))\}) \cup \{(\boldsymbol{x},u)\}$

Conjecture

$\|\boldsymbol{K}\|^{\mathscr{M},g} = \blacklozenge[\![\boldsymbol{K}]\!]^{\mathscr{M},g}$, for any 'decent' DRS $\boldsymbol{K}$, model $\mathscr{M}$, and g: $Fr(\boldsymbol{A}) \to \mathscr{U}_{\mathscr{M}}$

References

Kamp, Hans: 'A Theory of Truth and Semantic Representation'. In: J. Groenendijk *et al.* (eds.), *Formal Methods in the Study of Language. Part 1*. Amsterdam 1981. 277–322.

Kamp, Hans; Reyle, Uwe: *From Discourse to Logic*. Dordrecht 1993.